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An Adjustable Method for Data Ranking based on Fuzzy Soft Sets

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Abstract

This work deals with an adjustable approach for ranking objects based on fuzzy soft models. We first generate two preorder relations, thus, two equivalence relations based on the fuzzy soft topology. Then, a method for data ranking is designed according to these binary relations. The connection between Separation axioms and such data ranking method is also studied. Finally, an example is provided to illustrate this method for problem of data ranking.

Keywords: Decision Making, Fuzzy Soft Product Topology, Fuzzy Soft Subspace, Fuzzy Soft Topology, Preorder Relation

1. Introduction

The concept of soft set was appeared firstly at the title of Molodtsov's[1] article to cover complicated Problems with uncertainty that the prior mathematical tools cannot handled successfully. There have been several mathematical theories such as interval mathematics, probability theory, fuzzy set theory[2], and rough set theory[3] to deal with various types of uncertainty, imprecise and vagueness. However, as pointed out in[1], lack of parametrization tools can be seen as the main limitation shared by these theories. As a result, in 1999, Molodtsov[1] introduced a new mathematical approach named soft set theory to deal with uncertainty. A soft set over a universal set X is defined by a set-valued map $f : E \to 2^X$, called s-function, which describes the elements of X on the basis of parameter set E approximately[4].

Following the presentation of soft set by Molodtsov[1], Maji et al.[6] introduced a new hybrid notion so called fuzzy soft set by combining the theory of fuzzy set and the soft set theory. Moreover, some operations for fuzzy soft sets like union, intersection and complement were introduced by them. In[7], Kharal and Ahmad presented the notion of fuzzy soft mapping between two fuzzy soft spaces. Roy and Samanta[8] defined the concept of fuzzy soft topology as a topological structure over an ordinary set, called universal set, where this new topology is perceived as a collection of fuzzy soft sets over the universal set which is closed under arbitrary supremum and finite infimum; and contains absolute, and null fuzzy soft sets. Zahedi et al.[9] continued the work of Roy and Samanta and proposed the concept of fuzzy soft product topology and studied some of its properties. The concept of fuzzy soft boundary as well as some of its properties were also considered by them in[10].

Meanwhile, the application of fuzzy soft sets in other scientific fields has received much attention, especially in decision-making in which the problem of ranking and classification of objects can be seen as crucial issues to evaluate objects based on some parameters. There are a general method in the literature to solve decision-making problems based on (fuzzy) soft set theory. These existing techniques, mostly, focus on the number of parameters possessed by each object. The initial efforts to take into account the applicability of fuzzy soft set theory in

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decision-making can be found in the work of Roy and Maji[11] who defined the concept of score value, that equals the difference between total number of parameters in which an object dominates all other objects and is dominated by all other objects. The alternative with the maximum score value may be seen as an optimum object. Later, Kong et al.[12] modified this method by considering the concept of fuzzy choice value as the summation of membership degrees of an object regarding to all parameters. Feng et al.[13] considered the notion of α-level soft set by applying the concept of α-level sets in fuzzy set theory and used it to propose a new approach for solving a decision-making problem. By using the concept of choice value, firstly introduced in[5] as the total number of parameters possessed by each object, at any certain level of membership degrees the Optimum object can be selected as the object with the highest amount of choice value.

According to these methods, which are formulated by the choice value and the score value, objects are ranked based on a linear order, while in reality preorder relations and preference relations are mainly involved in a decision-making problem. On the other hand, it is well-known that topological structures and order structures have close relations. Any topology may generate a preorder relation over the underlying set and any preorder relation can generate a topology. This issue emphasizes the importance and necessity of this present paper whose main purpose is to develop a topological approach for ranking objects based on fuzzy soft models. To obtain this objective, our methodology includes two main phases: 1. Induce two topological structures based on the fuzzy soft topology, and 2. Generate some binary relationships in order to establish a method of ranking objects for choosing the best one. So, the organization of this paper is as follows. In Section 2, we review some basic concepts and definitions which will be used along this work. In Section 3, we discus our main results. We first initiate two induced topologies $\tau^{u}_{e,\alpha}$ and $\tau^{l}_{e,\beta}$ over the universal set X where the open sets of these induced topologies are understood as two following sets: the set of alternatives preferred to some other objects; and the set of alternatives to which some objects are preferred. Then, we present our main proposal for this research. We design a method for ranking objects based on two preorder relations associated with the topological spaces $\left(X,\tau^{u}_{e,\alpha}\right)$ and $\left(X,\tau^{l}_{e,\beta}\right)$ and consider the relationship between the Separation axioms and this data ranking method. Finally, in Section 4, an example is provided to illustrate our method.

2. Preliminaries

Throughout this paper, let X be the set of objects and E be the set of parameters. Let I^X, where $I = [0,1]$, denotes the set of all fuzzy subsets of X. The notation $\mathcal{F.S}(X, E)$ is used to show the collection of all fuzzy soft sets over the universal set X regarding to the parameter set E. P is used to show the negation of property P. We will also abbreviate the term "fuzzy soft" to "*FS*" in follows.

Definition 2.1.[6] *A pair (f, E), denoted by f_E, is called a FS-set over X if f is a mapping given by $f : E \to I^X$ where for any $e \in E$, $f(e)$ is a fuzzy subset of X with membership function $f_e : X \to [0,1]$.*

For two *FS*-sets f_E and g_E over the common universe X, the complement of f_E is denoted by f^c_E and is defied by $f^C : E \to I^X$ where $\forall e \in E, f^c_E = 1 - f_e$. The union of f_E and g_E, denoted by $f_E \tilde{\vee} g_E$, is the *FS*-set $(f \tilde{\vee} g)_E$ where $\forall_e \in E$ and $\forall_x \in X$, we have $(f \tilde{\vee} g)_e(x) = \max\{f_e(x), g_e(x)\}$. The intersection of f_E and g_E, denoted by $f_E \tilde{\wedge} g_E$, is the FS-set $(f \tilde{\wedge} g)_E$ where $\forall_e \in E$ and $\forall_x \in X$, we have $(f \tilde{\wedge} g)_e(x) = \min\{f_e(x), g_e(x)\}$. The null *FS*-set Φ is defined as a *FS*-set over X such that for each $e \in E$, $f_e(x) = 0$, $\forall x \in X$. And the absolute *FS*-set $\tilde{X}$ is defined as a *FS*-set over X where for all $e \in E$, $f_e(x) = 1$, $\forall x \in X$.

Definition 2.2.[8] *A FS-topological space is denoted by the triplet (X, E, τ) such as τ, named FS-topology, is a collection of FS-sets over X, so-called FS-open sets, closed under arbitrary supremum and finite infimum and containing absolute and null FS-sets.*

Definition 2.3.[9] *Let $\{(X_i, E_i, \tau_i) : i \in J\}$ be a family of* FS-*topological spaces. The FS-topology $\tau^{\otimes}$ generated by family $\left\{\left[\pi^{X,E}_{S}\right]^{-1}\left(f_{sE_s}\right) : f_{sE_s} \in \tau_s, s \in J\right\}$, as a FS-subbase, is called the FS-product topology where for any $f_{sE_s} \in \mathcal{F.S}\left(X_s, E_s\right)$, $\alpha \in \prod_i E_i$, and $x \in \prod_i X_i$, the FS-projection mapping*

$$\pi^{X,E}_{s} : \mathcal{F.S}\left(\prod_i X_i, \prod_i E_i\right) \to \mathcal{F.S}\left(X_s, E_s\right)$$

is defined as $\left[\pi^{X,E}_{s}\right]^{-1}\left(f_{sE_s}\right)(\alpha)(x) = f_s\left(\alpha_s\right)\left(x_s\right)$ such that for any $s \in J, \pi^X_s : \prod_i X_i \to X_s$ and $\pi^E_s : \prod_i E_i \to E_s$ are ordinary projection maps over X and E, respectively, and $\alpha_s \in E_s$, $x_s \in X_s$.

Definition 2.4.[9] *If (X, E, τ) is a FS-topological space, then the collection*

$$\tau_{Y,F} = \left\{ f_F^Y : f_E \in \tau,\ Y \subset X,\ F \subset E \right\}$$

is a FS-topology over $Y \subset X$ and called the FS-subspace topology where the FS-set $f_F^Y \in \mathcal{F}.\mathcal{S}(Y,F)$, named the restriction of FS-set $f_E \in \mathcal{F}.\mathcal{S}(X, E)$, is defined by mapping $f^Y : F \to I^Y$ such that for any $\epsilon \in F$ and $\forall_y \in Y$, $f_F^Y(y) = f_\epsilon(y)$.

3. Main Results

In this section, we discuss our main results.

3.1 α-upper-e and β-lower-e Topological Spaces

In this section, we discuss two induced topological structures determined by the α-level sets of a fuzzy binary relation $f : E \times X \to [0,1]$ generated by a *FS*-set f_E. A result concerning Molodtsov's soft sets proposed by Feng et al.[13] inspired us to provide the following theorem.

Theorem 3.1. *If f_E is a FS-set over X and $\tilde{R}$ is a fuzzy binary relation from E into X, then*

i. *f_E induces a fuzzy binary relation, denoted by $\tilde{R}^f$, from E into X.*
ii. *$\tilde{R}$ generates a FS-set, denoted by $f_E^{\tilde{R}}$, over X with respect to the set E.*

Moreover $f_E = f_E^{\tilde{R}^f}$ and $\tilde{R} = \tilde{R}^{f^{\tilde{R}}}$.

Proof. Consider function $\tilde{R}^f : E \times X \to I$ where $\tilde{R}^f(e,x) = f_e(x)$ for all $e \in E$ and $x \in X$. The theorem is derived immediately.

We will use the map $f : E \times X \to I$ where $f(e, x) = f_e(x)$ to show the induced fuzzy binary relation $\tilde{R}^f$ by *FS*-set f_E. The value of $f(e, x)$ can be perceived as the degree of relationship between parameter e and object x in terms of f. Thus, if there is no chance for confusion, the notation f_E will be replaced by f everywhere we need to represent the concept of *FS*-set as a fuzzy binary relation.

Motivated from the above theorem and regarding to the concept of strong α-level set in fuzzy set theory, we give the following definition.

Definition 3.1. *Let $x \in X$ and $e \in E$ be some arbitrary object and parameter, respectively. α-upper-e description set and β-lower-e description set are defined as below:*

$$U.Des_\alpha\left(e^f\right) = \left\{x \in X; f\left(e,x\right) > \alpha, \alpha \in [0,1)\right\} = \bigcup_{x \in f_e^{-1}(\alpha,1]} x = f_e^{-1}(\alpha,1]$$

$$L.Des_\beta\left(e^f\right) = \left\{x \in X; f\left(e,x\right) > \beta, \beta \in [0,1)\right\} = \bigcup_{x \in f_e^{-1}(0,\beta]} x = f_e^{-1}(0,\beta]$$

Proposition 3.1. *Let f and g be two fuzzy binary relations induced by FS-sets f_E and g_E, respectively. Then for any e, ε ∈ E, for all α ∈ [0,1), and β ∈ (0,1] we have*

i. *$U.Des_\alpha(e^f) \cap U.Des_\alpha(e^g) = U.Des_\alpha(e^{f \tilde{\wedge} g})$ and $U.Des_\alpha(e^f) \cup U.Des_\alpha(e^g) = U.Des_\alpha(e^{f \tilde{\vee} g})$.*
ii. *$L.Des_\beta(e^f) \cap L.Des_\beta(e^g) = L.Des_\beta(e^{f \tilde{\vee} g})$ and $L.Des_\beta(e^f) \cup L.Des_\beta(e^g) = L.Des_\beta(e^{f \tilde{\wedge} g})$.*
iii. *$U.Des_\alpha(e^f) \cap U.Des_\alpha(\varepsilon^g) = U.Des_\alpha(e^f \wedge \varepsilon^g)$ and $U.Des_\alpha(e^f) \cup U.Des_\alpha(\varepsilon^g) = U.Des_\alpha(e^f \vee \varepsilon^g)$.*
iv. *$L.Des_\beta(e^f) \cap L.Des_\beta(e^g) = L.Des_\beta(e^{f \vee g})$ and $L.Des_\beta(e^f) \cup L.Des_\beta(e^g) = L.Des_\beta(e^{f \wedge g})$.*
v. *If $f_E = \tilde{X}$, then $U.Des_\alpha(e^f) = X$ and $L.Des_\beta(e^f) = \emptyset$. Moreover if $f_E = \Phi$ then $U.Des_\alpha(e^f) = \emptyset$ and $L.Des_\beta(e^f) = X$.*

Proof. We just prove part (i). The similar technique is used to show the other parts.

$$\begin{aligned} z \in U.Des_\alpha\left(e^f\right) \cap U.Des\left(e^g\right) &\Leftrightarrow z \in f_e^{-1}(\alpha,1] \cap g_e^{-1}(\alpha,1] \\ &\Leftrightarrow f_e(z) > \alpha \wedge ge(z) > \alpha \\ &\Leftrightarrow \min \left\{f_e(z), ge(z)\right\} > \alpha \\ &\Leftrightarrow \left(f \tilde{\wedge} g\right)(e)(z) > \alpha \\ &\Leftrightarrow z \in \left(f \tilde{\wedge} g\right)_e^{-1}(\alpha,1] \Leftrightarrow z \in U.Des_\alpha\left(e^{f \wedge g}\right). \end{aligned}$$

Thus $U.Des_\alpha(e^f) \cap U.Des_\alpha(e^g) = U.Des_\alpha(e^{f \tilde{\wedge} g})$. Now let $z \in U.Des_\alpha(e^f) \cup U.Des_\alpha(e^g)$. Then

$$\begin{aligned} z \in U.Des_\alpha\left(e^f\right) \cup U.Des_\alpha\left(e^g\right) &\Leftrightarrow z \in f_e^{-1}(\alpha,1] \cup g_e^{-1}(\alpha,1] \\ &\Leftrightarrow f_e(z) > \alpha \vee ge(z) > \alpha \\ &\Leftrightarrow \max \left\{f_e(z), ge(z)\right\} > \alpha \\ &\Leftrightarrow \left(f \tilde{\vee} g\right)(e)(z) > \alpha \\ &\Leftrightarrow z \in \left(f \tilde{\vee} g\right)_e^{-1}(\alpha,1] \Leftrightarrow z \in U.Des_\alpha\left(e^{f \vee g}\right). \end{aligned}$$

This implies that $U.Des_\alpha(e^f) \cup U.Des_\alpha(e^g) = U.Des_\alpha(e^{f \tilde{\vee} g})$.

Theorem 3.2. *Let (X, E, τ) be a FS-topological space.*

- *The collection $\left\{f_e^{-1}(\alpha,1] : f \in \tau\right\}$, denoted by $\tau_{e,\alpha}^u$, is a topology over X that is called α-upper-e topology induced by FS-topology τ.*

- *The collection* $\mathcal{B}^{l}_{e,\beta}=\left\{f_e^{-1}[0,\beta):f\in\tau\right\}$ *is a base for a topology over X, denoted by* $\tau^{l}_{e,\beta}$ *and is called β-lower-e topology induced by FS-topology τ.*

Proof. i. (a) That $X, \emptyset \in \tau^{u}_{e,\alpha}$ implies from $\tilde{X}$ and Φ are in τ.

(b) Let $\{A_\lambda\}_{\lambda\in\wedge}\subset\tau^{u}_{e,\alpha}$, so for any $\lambda\in\wedge$, there exists $f_{\lambda E}\in\tau$ such that $A_\lambda=(f_\lambda(e))^{-1}(\alpha,1]$. It is easily to check that $\cup_\lambda A_\lambda=\cup_\lambda\left(f_\lambda(e)\right)^{-1}(\alpha,1]=\left(\left(\tilde{\vee}_\lambda f_\lambda\right)(e)\right)^{-1}(\alpha,1]$, that the later is a $\tau^{u}_{e,\alpha}$-open set since $\tilde{\vee}_\lambda f_\lambda\in\tau$.

(c) If A and B are some $\tau^{u}_{e,\alpha}$ open sets, then for some $f_E, g_E\in\tau$ we have $A=f_e^{-1}(\alpha,1]$ and $B=g_e^{-1}(\alpha,1]$. We have $A\cap B=f_e^{-1}(\alpha,1]\cap g_e^{-1}(\alpha,1]=((f\tilde{\wedge}g)(e))^{-1}(\alpha,1]$ where $((f\tilde{\wedge}g)(e))^{-1}(\alpha,1]\in\tau^{u}_{e,\alpha}$ implies from $f_E\tilde{\wedge}g_E\in\tau$.

ii. (a) That $X\in\mathcal{B}^{1}_{e,\beta}$ implies from Φ is in τ.

(b) Let A and B be in $\mathcal{B}^{1}_{e,\beta}$. There exist two τ-*FS*-open sets f_E, g_E such that $A=f_e^{-1}[0,\beta)$ and $B=g_e^{-1}[0,\beta)$ So, we have $A\cap B=f_e^{-1}[0,\beta)\cap g_e^{-1}[0,\beta)=((f\tilde{\vee}g)(e))^{-1}[0,\beta)\in\mathcal{B}^{1}_{e,\beta}$ implies from $f_E\tilde{\wedge}g_E\in\tau$.

It is easily to check that for any τ-*FS*-closed set f, $f_e^{-1}[1-\alpha,1]$ is a $\tau^{u}_{e,\alpha}$-closed set and $f_e^{-1}[0,1-\beta]$ is a $\tau^{l}_{e,\beta}$ closed set. Moreover, it is clear that if τ and γ are two different *FS*-topologies over X such that $\tau\subset\gamma$, then $\tau^{u}_{e,\alpha}\subset\gamma^{u}_{e,\alpha}$ and $\tau^{l}_{e,\beta}\subset\gamma^{l}_{e,\beta}$.

Theorem 3.3. Let $\{(X_i, E_i, \tau_i): i\in J\}$ *be a family of FS-topological spaces and* $(\prod_i X_i, \prod_i E_i, \tau^{\otimes})$ *indicates the respective FS-product topological space. Then for any* $e\in\prod_i E_i$, *we have* $\prod_i\tau^{u}_{ie_i,\alpha}=\tau^{\otimes u}_{e,\alpha}$ *and* $\prod_i\tau^{l}_{ie_i,\beta}=\tau^{\otimes l}_{e,\beta}$ *where* $\prod_i\tau^{u}_{ie_i,\alpha}$ *and* $\prod_i\tau^{l}_{ie_i,\beta}$, *respectively, show the product topology of the topological spaces* $\left(X_i\tau^{u}_{ie_i,\alpha}\right)$ *and* $\left(X_i\tau^{l}_{ie_i,\beta}\right)$.

Proof. It suffices to show that the subbases of the topologies $\prod_i\tau^{l}_{ie_i,\beta}$ and $\tau^{\otimes l}_{e,\beta}$ are the same collections. Assume that $\left(\pi_s^{X,E}\right)^{-1}\left(f_{sE_s}\right)$ be a member of subbase of the *FS*-product topology $\tau^{\otimes}$ where $f_{sE_s}\in\tau_s$, that is, $\left(\left[\pi_s^{X,E}\right]^{-1}\left[f_s\right]\left[\left(e_i\right)\right]\right)^{-1}(\alpha,1]\in\tau^{\otimes u}_{e,\alpha}$. By Definition 2.3 we have $\left(\left[\pi_s^{X,E}\right]^{-1}\left[f_s\right]\left[\left(e_i\right)_i\right]\right)^{-1}(\alpha,1]=\pi_s^{X^{-1}}\left(f_{s_{e_s}}{}^{-1}(\alpha,1]\right)$, where $f_s\in\tau_s$, $e_s\in E_s$, and $f_{s_{e_s}}{}^{-1}(\alpha,1]$ is a $\tau^{u}_{ie_i,\alpha}$-open set. Thus, any typical open set in $\tau^{\otimes u}_{e,\alpha}$ can be seen as an $\tau^{u}_{ie_i,\alpha}$-open set and vice versa. Hence, $\prod_i\tau^{u}_{ie_i,\alpha}=\tau^{\otimes u}_{e,\alpha}$.

Second part is obtained by the similar technique.

Theorem 3.4. *Let* (X, E, τ) *be a FS-topological space and* $(Y, F, \tau_{Y,F})$ *be the respective FS-subspace topology. Then for any* $e\in F\subset E$ *we have* $\tau^{u}_{e,\alpha Y}=\tau_{Y,F^{u}_{e,\alpha}}$ *and* $\tau^{l}_{e,\beta Y}=\tau_{Y,F^{l}_{e,\beta}}$ *where* $\tau^{u}_{e,\alpha Y}$ *and* $\tau^{l}_{e,\beta Y}$, *respectively, show the subspace topology of the topological spaces* $(X,\tau^{u}_{e,\alpha})$ *and* $(X,\tau^{l}_{e,\beta})$.

Proof. Assume that $V\in\tau_{Y,F^{u}_{e,\alpha}}$ and $e\in F\subset E$. So that, there exists $f_E\in\tau$ such that $V=f_e^{Y-1}(\alpha,1]$. By Definition 2.4 we have $f_e^{Y-1}(\alpha,1]=f_e^{-1}(\alpha,1]\cap Y$ where the latter is an open set in the subsapce topology $\tau^{u}_{e,\alpha Y}$. This shows $\tau^{u}_{e,\alpha Y}=\tau_{Y,F^{u}_{e,\alpha}}$. The second part is implied similarly.

3.2 α-upper-*e* and, β-lower-*e* Preordered Relations

α-upper-*e* topology and β-lower-*e* topology create two preorder relations and, thus, two equivalence relations over the universal set X as follow.

Theorem 3.5. *Let* (X, E, τ) *be a FS-topological space,* $\alpha\in[0,1)$ *and* $\beta\in(0,1]$.

i. The binary relation $\succsim^{\tau}_{e,\alpha}$ *on X that is defined as*

$$y\succsim^{\tau}_{e,\alpha}x\Leftrightarrow\left[\forall V\in\tau^{u}_{e,\alpha}:x\in V\Rightarrow y\in V\right] \qquad (1)$$

is a preorder relation (a reflexive and transitive relation), called α-upper-e preorder relation, on X. We say that y is at least as good as x with respect to parameter e and value α.

ii. The binary relation $\precsim^{\tau}_{e,\beta}$ *on X that is defined as*

$$y\precsim^{\tau}_{e,\beta}x\Leftrightarrow\left[\forall U\in\tau^{l}_{e,\beta}:x\in U\Rightarrow y\in U\right] \qquad (2)$$

is a preorder relation, called β-lower-e preorder relation, on X. We say that x is at most as worst as y with respect to parameter e and value β.

Proof. It is derived from Theorem 3.2.

Proposition 3.2. *Let* (X, E, τ) *be a FS-topological space. Then*

i. *Let $N^u_{e,\alpha}(x)$ and $N^u_{e,\alpha}(y)$ be the respective collections of all $\tau^u_{e,\alpha}$-neighborhoods of x and y. Then $y \succsim^{\tau}_{e,\alpha} x$ iff $N^u_{e,\alpha}(x) \subset N^u_{e,\alpha}(y)$. Similarly, if $N^l_{e,\beta}(x)$ and $N^l_{e,\beta}(y)$ are the collections of all $\tau^l_{e,\beta}$-neighborhoods of* x and y, *respectively, then $y \precsim^{\tau}_{e,\beta} x$ iff $N^u_{e,\alpha}(x) \subset N^u_{e,\alpha}(y)$.*

ii. *$y \succsim^{\tau}_{e,\alpha} x$ iff $x \in \tau^u_{e,\alpha}$-Cl{y} and $y \precsim^{\tau}_{e,\beta} x$, iff $x \in \tau^l_{e,\beta}$-Cl{y} where $\tau^u_{e,\alpha}$-Cl{y} and $\tau^l_{e,\beta}$-Cl{y} indicate the respective closure of singleton {y}* in *$(X \tau^u_{e,\alpha})$ and $(X \tau^l_{e,\beta})$*[11]

iii. *If γ is finer than τ, then $y \succsim^{\gamma}_{e,\alpha} x$ implies $y \succsim^{\tau}_{e,\alpha} x$ and $y \precsim^{\gamma}_{e,\beta} x$ implies $y \precsim^{\tau}_{e,\beta} x$.*

iv. *If $\tau^u_{\epsilon,\alpha}$ is finer than $\tau^u_{e,\alpha}$, then $y \succsim^{\tau}_{\epsilon,\alpha} x \Rightarrow y \succsim^{\tau}_{e,\alpha} x$ and if $\tau^l_{\epsilon,\beta}$ is finer than $\tau^l_{e,\beta}$, then $y \precsim^{\tau}_{\epsilon,\beta} x \Rightarrow y \precsim^{\tau}_{e,\beta} x$.*

v. *if $0 \le \alpha < \inf_{f \in \tau} f_e(x)$, then for any $x \in X$ $x \succsim^{\tau}_{e,\alpha} y$, $\forall y \in X$. And if $\sup_{f \in \tau} f_e(x) < \beta \le 1$, then for any $x \in X$ $x \precsim^{\tau}_{e,\beta} y$, $\forall y \in X$.*

vi. *If $\tau^u_{e,\alpha} = \{X, \emptyset\}$, then $\forall x, y \in X$ $x \succsim^{\tau}_{e,\alpha} y$. Similarly, if $\tau^l_{e,\beta} = \{X, \emptyset\}$, then $x \precsim^{\tau}_{e,\beta} y$ $\forall x, y \in X$.*

vii. *If $\tau = \{\tilde{X}, \Phi\}$, then for any $\alpha \in [0,1)$, $\beta \in (0,1]$ and $\forall x, y \in X$, $x \succsim^{\tau}_{e,\alpha} y$ and $x \precsim^{\tau}_{e,\beta} y$.*

Proof. It is followed from Theorem 3.5.

Theorem 3.6. *Let (X, E, τ) be a finite FS-topological space.*

i. *$(X, \succsim^{\tau}_{e,\alpha})$ is a partially ordered set iff $(X, \tau^u_{e,\beta})$ is a T_0 space.*

ii. $(X, \precsim^{\tau}_{e,\beta})$ is a partially ordernd set iff $(X, \tau^l_{e,\beta})$ is a T_0 space.

Proof. It is followed from Theorem 3.5.

Theorem 3.7. *Let (X, E, τ) be a finite FS-topological space.*

i. *The set $(\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha, 1]$, if it is nonempty, is the maximal set of the preordered set $(X, \succsim^{\tau}_{e,\alpha})$.*

ii. *The set $(\tilde{\vee}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}[0, \beta)$ is the minimal set of the preordered set $(X, \precsim^{\tau}_{e,\beta})$.*

Proof. i. Let (X, E, τ) be a finite *FS*-topological space and $f_\lambda \in \tau$ where $\lambda \in \Lambda$ is an indexing set. For some $e \in E$, let $x^* \in (\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha, 1]$, then $\forall \lambda$ e Λ, $x^* \in (f_\lambda(\mathrm{e}))^{-1}(\alpha,1]$. If y is an arbitrary element of X and V is a $\tau^u_{e,\alpha}$-open set containing y, then there exists $f_E \in \tau$ such that $y \in V = f_e^{-1}(\alpha,1]$. Thus, $x^* \in f_e^{-1}(\alpha,1] = V$ implies that $x^* \succsim^{\tau}_{e,\alpha} y$. On the other hand, if there exists $z \in X$ where $z \notin (\tilde{\wedge}_{\lambda \in \Lambda}(f_\lambda(\mathrm{e}))^{-1}(\alpha,1]$ but for all $y \in X$, $z \succsim^{\tau}_{e,\alpha} y$, then $z \succsim^{\tau}_{e,\alpha} x^*$. So, for all $\lambda \in \Lambda$, $z \in (f_\lambda(\mathrm{e}))^{-1}(\alpha,1]$. Therefore, $z \in \bigcap_\lambda (f_\lambda(\mathrm{e}))^{-1}(\alpha,1] = (\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha,1]$ and this is a contradiction.

ii. It follows the similar technique.

Theorem 3.8.

i. *If $\{(X, \tau^u_{e,\alpha})$ is a T_1 space, then $(\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha,1] = \emptyset$.*

ii. *If $(X, \tau^l_{e,\beta})$ is a T_1 space, then $(\tilde{\vee}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}[0,\beta) = \emptyset$.*

Proof. i. Suppose that $(X, \tau^u_{e,\alpha})$ be a T_1 space and $(\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha,1] = \emptyset$. Take $x \in (\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha, 1]$, and $y \in X$ such that $x \neq$ y. Then there exist $\tau^u_{e,\alpha}$-open sets V and U containing y and x, respectively such that $y \notin U$ and $x \notin V$. But this is a contradiction with $x \in (\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha, 1]$. Hence $(\tilde{\wedge}_{\lambda \in \Lambda} f_\lambda(\mathrm{e}))^{-1}(\alpha,1] = \emptyset$.

ii. It is similar to i.

Theorem 3.9. *Let $\{(X_i, E_i, \tau_i) : i \in J\}$ be a family of FS-topological space and, $(\Pi_i X_i, \Pi_i E_i, \tau^{\otimes})$ indicates FS -product topological space. Then*

i. *$x \succsim^{\tau^{\otimes}}_{e,\alpha} y$ iff $x_s \succsim^{\tau_s}_{e_s,\alpha} y_s$,*

ii. *$x \precsim^{\tau^{\otimes}}_{e,\beta} y$ iff $x_s \precsim^{\tau_s}_{e_s,\beta} y_s$.*

where $e \in \prod_i E_i$ and $x, y \in \prod_i X_i$, such that for any $s \in J$, $e_s \in E_s$ and $x_s, y_s \in X_s$.

Proof. i. First assume that $x \succsim^{\tau^{\otimes}}_{e,\alpha} y$. Let V be an $\tau^u_{se_s,\alpha}$-open set in X_s where $y_s \in V$. So there exists a τ-*FS*-open set f_s such that $V = f_{s_{e_s}}^{-1}(\alpha,1]$. Consider $y \in \Pi_i X_i$ where y_s be its s-th coordinate, then $y \in \pi_s^{X-1}(V) = ([\pi_s^{X,E}]^{-1}[f_s][(e_i)])^{-1}(\alpha,1]$. So $x \in \pi_s^{X-1}(V)$, and hence $x_s \in f_{s_{e_s}}^{-1}(\alpha, 1]$. This shows

$x_s \succsim^{\tau_s}_{e_s,\alpha} y_s$. Conversely, let $e \in \Pi_i E_i$ and $x = (x_i)$, $y = (y_i) \in \Pi_i X_i$ and assume U be an $\tau^{\otimes u}_{e,\alpha}$-open set in $\Pi_i X_i$ where $y \in U$. Then there exists a τ_s-FS-open set f_s such that $U = ([\pi_s^{X|,E}]^{-1}[f_s][(e_i)])^{-1}(\alpha, 1]$ and $y \in ([\pi_s^{X,E}]^{-1}[f_s]\ [(e_i)])^{-1}(\alpha, 1]$. So $y_s \in f^{-1}_{s_{e_s}}(\alpha, 1]$ for any $s \in J$ and then $x_s \in f^{-1}_{s_{e_s}}(\alpha, 1]$, hence $x \in ([\pi_s^{X,E}]^{-1}[f_s][(e_i)])^{-1}(\alpha, 1]$. This completes the proof.

ii. It follows the similar technique.

Theorem 3.10. *Let (X, E, τ) be a FS-topological space and $(Y, F, \tau_{Y,F})$ be the respective FS-subspace topology where $Y \subset X$ and $F \subset E$. Then for any $x,y \in Y \subset X$ and $e \in F \subset E$,*

i. $x \succsim^{\tau_{Y,F}}_{e,\alpha} y$ *iff* $x \succsim^{\tau}_{e,\alpha} y$,

ii. $x \precsim^{\tau_{Y,F}}_{e,\beta} y$ *iff* $x \precsim^{\tau}_{e,\beta} y$.

Proof. i. First assume that $x \succsim^{\tau_{Y,F}}_{e,\alpha} y$. Take $e \in F \subset E$, x, $y \in Y \subset X$ and let V be an $\tau^{u}_{e,\alpha}$-open set in X where $y \in V$. There exists $f \in \tau$ such that $y \in f_e^{-1}(\alpha, 1]$, thus, $y \in (f_e^Y)^{-1}(\alpha, 1]$ since $y \in Y \cap f_e^{-1}(\alpha, 1]$. So $x \in (f_e^Y)^{-1}(\alpha, 1]$ is followed from $x \succsim^{\tau_{Y,F}}_{e,\alpha} y$ Hence $x \in f_e^{-1}(\alpha, 1]$ which shows $x \succsim^{\tau}_{e,\alpha} y$. Conversely, let $x \succsim^{\tau}_{e,\alpha} y$. If W be an $\tau_{Y,F^u_{e,\alpha}}$-open set in Y where $y \in W$. Then there exists $f \in \tau$ such that $y \in (f_e^Y)^{-1}(\alpha, 1]$, thus, $y \in f_e^{-1}(\alpha, 1]$ since $y \in Y \cap f_e^{-1}(\alpha, 1]$. Consequently, $x \in f_e^{-1}(\alpha, 1]$ and so $y \in (f_e^Y)^{-1}(\alpha, 1]$. This shows that $x \succsim^{\tau_{Y,F}}_{e,\alpha} y$.

ii. It follows the similar technique.

Theorem 3.11. *Let (X, E, τ) be a FS-topological space.*

i. *The binary relation $\simeq^{\tau}_{e,\alpha}$ that is defined on X as*

$$y \simeq^{\tau}_{e,\alpha} x \Leftrightarrow [y \succsim^{\tau}_{e,\alpha} x, x \succsim^{\tau}_{e,\alpha} y] \quad (3)$$

is an equivalence relation over X. The equivalence relation $\simeq^{\tau}_{e,\alpha}$ generates the partition $P^{\tau}_{e,\alpha}$ of X where the equivalence classes are defined by $[x]^{\tau}_{e,\alpha} = \{z \in X : z \simeq^{\tau}_{e,\alpha} x\}$ and called α-upper-e equivalence classes.

ii. *The binary relation $\simeq^{\tau}_{e,\alpha}$ that is defined on X as*

$$y \simeq^{\tau}_{e,\beta} x \Leftrightarrow [y \precsim^{\tau}_{e,\beta} x, x \precsim^{\tau}_{e,\beta} y] \quad (4)$$

is an equivalence relation over X. The partition $P^{\tau}_{e,\beta}$ of X is generated by the equivalence relation $\simeq^{\tau}_{e,\alpha}$ where the equivalence classes are defined by $[x]^{\tau}_{e,\beta} = \{z \in X : z \simeq^{\tau}_{e,\beta} x\}$ and called the β-lower-e equivalence classes.

Proof. It is followed from Theorem 3.5.

The equivalence relations $\simeq^{\tau}_{e,\alpha}$ and $\simeq^{\tau}_{e,\beta}$ partition the set X into disjoint classes providing a parametrizied collection of partitions of X.

Proposition 3.3. *Let (X, E, τ) be a FS-topological space. Then*

i. $y \simeq^{\tau}_{e,\alpha} x$ *iff* $N^u_{e,\alpha}(x) = N^u_{e,\alpha}(y)$ *and* $y \simeq^{\tau}_{e,\beta} x$ *iff* $N^u_{e,\alpha}(x) = N^u_{e,\alpha}(y)$.

ii. $y \simeq^{\tau}_{e,\alpha} x$ *iff* $\tau^u_{e,\alpha}$*-Cl*$\{x\}$ = $\tau^u_{e,\alpha}$*-Cl*$\{y\}$ *and* $y \simeq^{\tau}_{e,\beta} x$, *iff* $\tau^l_{e,\beta}$*-Cl*$\{x\}$ = $\tau^l_{e,\beta}$*-Cl*$\{y\}$.

iii. *If x and y are the maximal elements of the preordered set $(X, \succsim^{\tau}_{e,\alpha})$, then $x \simeq^{\tau}_{e,\alpha} y$ and if x and y are the minimal elements of the preordered set $(X, \precsim^{\tau}_{e,\beta})$ we have $x \simeq^{\tau}_{e,\beta} y$*

iv. *If γ is finer than τ, then $\simeq^{\gamma}_{e,\alpha}$ and $\simeq^{\gamma}_{e,\beta}$ are finer relations than $\simeq^{\tau}_{e,\alpha}$ and $\simeq^{\tau}_{e,\beta}$, respectively.*

v. $\simeq^{\tau}_{\epsilon,\alpha}$ *is a finer relation than* $\simeq^{\tau}_{e,\alpha}$ *if* $\tau^u_{\epsilon,\alpha}$ *is finer than* $\tau^u_{e,\alpha}$.

vi. *If* $\tau^l_{\epsilon,\beta}$ *is finer than* $\tau^l_{e,\beta}$ *then* $\simeq^{\tau}_{\epsilon,\beta}$ *is a finer relation than* $\simeq^{\tau}_{e,\beta}$.

vii. *If $0 \le \alpha < \inf_{f\in\tau} f_e(x)$, then $P^{\tau}_{e,\alpha}$ is the trivial partition $\{X\}$. And $P^{\tau}_{e,\beta}$ is the trivial partition $\{X\}$ if $\sup_{f\in\tau} f_e(x) < \beta \le 1$.*

viii. *If $\tau^u_{e,\alpha} = \{X, \emptyset\}$ the $P^{\tau}_{e,\alpha}$ is the trivial partition $\{X\}$. Similarly, If $\tau^l_{e,\beta} = \{X, \emptyset\}$ then $P^{\tau}_{e,\beta}$ is the trivial partition $\{X\}$.*

ix. *If $\tau = \{\tilde{X}, \Phi\}$, then for any $\alpha \in [0,1)$ and $\beta \in (0,1]$, $P^{\tau}_{e,\alpha}$ and $P^{\tau}_{e,\beta}$ are the trivial partition $\{X\}$.*

Proof. It is followed from Theorem 3.11 and Proposition 3.2.

Theorem 3.12. *Let $\{(X_i, E_i, T_i) : i \in J\}$ be a family of FS-topological spaces and $(\Pi_i X_i, \Pi_i E_i, \tau^{\otimes})$ indicates the respective FS-product topological space. Then*

i. $x \simeq^{\tau^{\otimes}}_{e,\alpha} y$ iff $x_s \simeq^{\tau_s}_{e_s,\alpha} y_s$,

ii. $x \simeq^{\tau^{\otimes}}_{e,\beta} y$ iff $x_s \simeq^{\tau_s}_{e_s,\beta} y_s$.

where $e \in \Pi_i E_i$ and $x, y \in \Pi_i X_i$ such that for any $s \in J$, $e_s \in E_s$ and $x_s, y_s \in X_s$.

Proof. It is followed from Theorem 3.9.

Theorem 3.13. *Let (X, E, τ) be a FS-topological space and $(Y, F, \tau_{Y,F})$ be the respective FS-subspace where $Y \subset X$ and $F \subset E$. Then for any $x, y \in Y \subset X$ and $e \in F \subset E$,*

i. $x \simeq^{\tau_{Y,F}}_{e,\alpha} y$ iff $x \simeq^{\tau}_{e,\alpha} y$,

ii. $x \simeq^{\tau_{Y,F}}_{e,\beta} y$ iff $x \simeq^{\tau}_{e,\beta} y$.

Proof. It is followed from Theorem 3.10.

Theorem 3.14. *Let (X, E, τ) be a FS-topological space. Then*

i. $(X, \tau^{u}_{e,\alpha})$ is T_1 iff $y \succsim^{\tau}_{e,\alpha} x \Leftrightarrow y = x$.

ii. $(X, \tau^{l}_{e,\beta})$ is T_1 iff $y \precsim^{\tau}_{e,\beta} x \Leftrightarrow y = x$.

Proof. Let X be a $\tau^{u}_{e,\alpha}$-T_1 space and x, y be two distinct points in X such that $y \succsim^{\tau}_{e,\alpha} x$. So for any $\tau^{u}_{e,\alpha}$-open set U containing x we have $y \in U$, but this is a contradiction with T_1 condition. Therefore, if $y \succsim^{\tau}_{e,\alpha} x.$, then $y = x$. To prove converse, take two distinct points x and y in X. By assumption we have $\neg y \succsim^{\tau}_{e,\alpha} x$ and $\neg x \succsim^{\tau}_{e,\alpha} y$. This implies that there exist $\tau^{u}_{e,\alpha}$-open sets V and W in X containing x and y, respectively, such that $y \notin V$ and $x \notin W$. This completes the proof.
Part (ii) is derived simiilarly.

Table 1. Tabular representation of f_1

f_1	e_1	e_2	e_3	e_4
c_1	0.2	0.3	0.6	0.6
c_2	0.4	0.3	0.7	0.5
c_3	0.8	0.9	0.9	0.7
c_4	0.7	0.6	0.6	0.5
c_5	0.4	0.7	0.5	0.8
c_6	0.5	0.1	0.7	0.7
c_7	0.3	0.2	0.9	0.5

Table 2. Tabular representation of f_2

f_1	e_1	e_2	e_3	e_4
c_1	0.1	0.4	0.4	0.7
c_2	0.2	0.2	0.6	0.4
c_3	0.9	0.8	0.7	0.6
c_4	0.7	0.5	0.6	0.6
c_5	0.3	0.7	0.4	0.7
c_6	0.2	0.2	0.5	0.8
c_7	0.5	0.2	0.8	0.7

4. An Applicable Example

The preorder relations $\succsim^{\tau}_{e,\alpha}$ and $\succsim^{\tau}_{e,\alpha}$; and, thus, the equivalence relations $\simeq^{\tau}_{e,\alpha}$ and $\simeq^{\tau}_{e,\beta}$ give ordering structures, which are not necessary linear relations, over the set of objects. In order to elaborate this concept consider the following example.

Example 4.1 Suppose that the mathematics department in the university X wants to fill the postdoctoral Position. There are 7 candidates who have applied for this position. The set of candidates is denoted by $C = \{c_1, c_2, \ldots, c_7\}$ which characterized by the set of parameters $E = \{e_1, e_2, e_3, e_4\}$. For $i = 1, 2, 3, 4$, the parameters e_k stand for "number of publication", "number of conferences attending in", "skilled foreign language" and "quality of research proposal", respectively. Let there are three professors from the mathematics department who decide about the candidates. After considering the resume of each candidates, the professors construct the following three *FS-sets* f_L $(i = 1, 2, 3)$ which are presented in the tabular form of FS-sets in Tables 1-3.

If decision makers want to rank the candidates based on the parameter e_2, then by assumption $\alpha = 0.69$ and $\beta = 0.31$, we have the following:

$$\tau^{u}_{e_2,0.69} = \{\emptyset, C, \{c_3, c_4, c_5\}, \{c_3, c_5\}\}$$

and

$$\tau^{l}_{e_2,0.31} = \{\emptyset, C, \{c_2, c_6\}, \{c_2, c_6, c_7\}, \{c_1, c_2, c_6, c_7\}\}$$

Table 3. Tabular representation of f_3

f_3	e_1	e_2	e_3	e_4
c_1	0.3	0.5	0.5	0.6
c_2	0.4	0.1	0.4	0.5
c_3	0.9	0.7	0.8	0.7
c_4	0.8	0.7	0.4	0.8
c_5	0.5	0.8	0.5	0.9
c_6	0.1	0.1	0.7	0.6
c_7	0.3	0.4	0.9	0.6

For any $1 \le i \le 7$, $c_3 \succsim^{\tau}_{e_2,0.69} c_i$ *and* $c_5 \succsim^{\tau}_{e_2,0.69} c_i$*. Furthermore,* $c_3 \simeq^{\tau}_{e_2,0.69} c_5$, $c_4 \simeq^{\tau}_{e_2,0.69} c_4$, *and* $c_i \simeq^{\tau}_{e_2,0.69} c_j$ *for all* $i, j \neq 3, 4, 5$.

For $1 \le i \le 7$, $c_2 \precsim^{\tau}_{e_2,0.31} c_i$ *and* $c_6 \precsim^{\tau}_{e_2,0.31} c_i$*. For all* $i \neq 2, 6$, $\precsim^{\tau}_{e_2,0.31} c_i$ *and for* $i \neq 2, 6, 7$ $c_i \precsim^{\tau}_{e_2,0.31} c_i$.

In addition, $c_2 \simeq^{\tau}_{e_2,0.31} c_6$, $c_7 \simeq^{\tau}_{e_2,0.31} c_7$, *and* $c_1 \simeq^{\tau}_{e_2,0.31} c_1$*. For* $i,j \neq 1, 2, 6, 7$, $c_i \simeq^{\tau}_{e_2,0.31} c_j$.

The following partitions of C *will be obtained*:

$$P^{\tau}_{e_2,0.69} = \{\{c_3, c_5\}, \{c_4\}, \{c_1, c_2, c_6, c_7\}\}$$

and

$$P^{\tau}_{e_2,0.31} = \{\{c_2, c_6\}, \{c_7\}, \{c_1\}, \{c_3, c_4, c_5\}\}$$

Note that the equivalence classes $\{c_3, c_5\}$ *and* $\{c_1, c_2, c_6, c_7\}$ *of* $P^{\tau}_{e_2,0.69}$ *are, respectively, open and closed sets in the upper topology* $\tau^{\upsilon}_{e_2,0.69}$*, while the block* $\{c_4\}$ *is not nor open neither closed set.*

Similarly, the equivalence classes $\{c_3, c_4, c_5\}$ *and* $\{c_2, c_6\}$ of $\tau^{l}_{e_2,0.31}$ *are, respectively, closed and open sets in the lower topology* $\tau^{l}_{e_2,0.31}$*, while the equivalence classes* $\{c_1\}$ *and* $\{c_7\}$ *are not nor open neither closed set.*

The below diagrams show the relationships among the candidates.

Figure 1 expresses the relations $\succsim^{\tau}_{e_2,0.69}$ *and* $\simeq^{\tau}_{e_2,0.69}$*, both, over the alternatives where* c_i*'s,* $1 \le i \le 7$*, stand for any candidates and arrow going from alternative* c_i *to alternative* c_j *shows that* $c_i \succsim^{\tau}_{e_2,0.69} c_j$*, which means alternative* c_i *is preferred to alternative* c_j*. Being in the same block shows* $c_i \simeq^{\tau}_{e_2,0.69} c_j$ *or* c_i *is equally preferable to* c_j*.*

Figure 2 expresses the both relations $\precsim^{\tau}_{e_2,0.31}$ *and* $\simeq^{\tau}_{e_2,0.69}$ *over the alternatives* c_i*'s,* $1 \le i \le 7$*. Arrow going from alternative* c_j *to alternative* c_j *shows that* $c_i \precsim^{\tau}_{e_2,0.31} c_j$ *and being in the same block shows* $c_i \simeq^{\tau}_{e_2,0.69} c_j$*.*

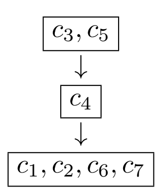

Figure 1. Diagram of Partition $P_{e_2,0.69}$

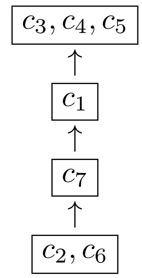

Figure 2. Diagram of Partition $P^{e_2,0.31}$

5. Conclusion

The importance of topology as a tool in preference theory is what motivates this study in which we characterize FS. We focus on two topological spaces $(X, \tau^{u}_{e,\alpha})$ and $(X, \tau^{l}_{e,\beta})$ referring to α-upper-e topology and β-lower-e topology that are induced based on the FS-topology τ. Then, two preorder relations, which are not necessarily totally ordered structure, are generated to design a method for ranking data. This paper also covers how a certain numbers of Separation axioms in the topological spaces $(X, \tau^{u}_{e,\alpha})$ and $(X, \tau^{l}_{e,\beta})$ affect these two preorder relations and consequently our proposed method for data ranking. We also present some example to show the results of this study are suitable for ranking data based on fuzzy soft models. The ordered structures *FS* the decision-making problems will be discussed in the future.

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7. References

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