Indian Journal of Science and Technology, *Vol 8(18), DOI: 10.17485/ijst/2015/v8i18/65394, August 2015*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Soft Strongly *g*-Closed Sets

K. Kannan[1*], D. Rajalakshmi[1] and R. Srikanth[2]

[1]Department of Mathematics, Srinivasa Ramanujan Centre, SASTRA University, Kumbakonam - 612 001, Tamil Nadu, India; anbukkannan@rediffmail.com, drajilak@gmail.com
[2]Department of Mathematics, SASTRA University, Thanjavur - 613 401, Tamil Nadu, India; srikanth@maths.sastra.edu

Abstract

Background/Objectives: The objective of the present paper is to define soft strongly *g*-closed sets and soft strongly *g*-open sets in soft topological spaces and study their basic properties. **Methods:** Here we used the concept of soft closure of soft interior and soft open sets to define soft strongly *g*-closed sets. **Findings:** The relationship between soft strongly *g*-closed sets and other existing sets has been investigated. Further the union, intersection of two soft strongly *g*-closed sets have been obtained. The authors discussed the complements of soft strongly *g*-closed sets in the last section. **Conclusion/ Improvements:** In future, the varieties of new continuous mappings and separation axioms based on these sets may be introduced and the future research may be undertaken in this direction.

Keywords: Soft Strongly *g*-closed Sets, Soft Strongly *g*-open Sets, Soft *g*-closed Sets, Soft *g*-open Sets, Soft *rg*-closed Sets, Soft *rg*-open Sets

1. Introduction

The concept of soft set was introduced by Molodtsov[1] as a new mathematical tool to overcome the difficulties of fuzzy sets[2], intuitionistic fuzzy sets[3], vague sets, interval mathematics[4,5] and rough sets[6]. Maji et al.[7,8] found an application of soft sets in decision making problems whereas Chen[9] gave a new idea of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory. Further, soft sets are a class of special information[10].

Muhammad Shabir and Munazza Naz introduced soft topological spaces and investigated their basic properties[11]. Meanwhile, generalized closed sets[12] in topological spaces were introduced by Levine in 1970 and one can see its development[13] and *g*-closed set is extended to soft topological spaces in the year 2012[14] and studied some more properties by Yiiksel et al.[15]. Soft semi star generalized closed sets[16] is introduced by Kannan and Rajalakshmi in 2015. Pauline Mary Helen et al.[17] introduced strongly *g*-closed sets in topological spaces. A set A is a strongly g-closed set if $cl[int(A)] \subseteq U$ whenever $A \subseteq U$ and U is open in X. In the present paper, we define the notions of soft strongly *g*-closed sets and soft strongly *g*-open sets in soft topological spaces and study their basic properties.

2. Preliminaries

Let U, E be an initial universe and a set of parameters, respectively. Let $P(U)$, E_1 denote the power set of U and a non-empty subset of E, respectively. A pair (M, E_1) is called a soft set over U, where $M : E_1 \to P(U)$ is a mapping. For $\varepsilon \in E_1$, $M(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (M, E_1). For two soft sets (M, E_1) and (N, E_2) over a common universe U, we say that (M, E_1) is a soft subset of (N, E_2) if (1) $E_1 \subseteq E_2$ and (2) for all $e \in E_1$, $M(e)$ and $N(e)$ are identical approximations. We write $(M, E_1) \tilde{\subseteq} (N, E_2)$. (M, E_1) is said to be a soft superset of (N, E_2), if (N, E_2) is a soft subset of (M, E_1). We denote it by $(M, E_1) \tilde{\supseteq} (N, E_2)$. Two soft sets (M, E_1) and (N, E_2) over a common universe U are said to be soft equal if (M, E_1) is a soft subset of (N, E_2) and (N, E_2) is a soft subset of (M, E_1).

The union of two soft sets of (M, E_1) and (N, E_2) over the common universe U is the soft set (O, E_3), where $E_3 = E_1 \cup E_2$ and for all $e \in E_3$, $O(e) = M(e)$ if $e \in E_1 - E_2$, $N(e)$ if

*Author for correspondence

$e \in E_2 - E_1$ and $M(e) \cup N(e)$ if $e \in E_1 \cap E_2$. We write $(M, E_1) \tilde{\cup} (N, E_2) = (O, E_3)$. The intersection (O, E_3) of two soft sets (M, E_1) and (N, E_2) over a common universe U, denoted by $(M, E_1) \tilde{\cap} (N, E_2)$, is defined as $E_3 = E_1 \cap E_2$, and $O(e) = M(e) \cap N(e)$ for all $e \in E_3$. The relative complement of a soft set (M, E_1) is denoted by $(M, E_1)'$ and is defined by $(M, E_1)' = (M', E_1)$ where $M' : E_1 \to P(U)$ is a mapping given by $M'(e) = U - M(e)$ for all $e \in E_1$.

Let X be an initial universe set, E be the set of parameters. Let τ be the collection of soft sets over X, then is said to be a soft topology on X if (1) $\tilde{\phi}$ $\tilde{X}$ belong to τ, (2) the union of any number of soft sets in τ belongs to τ, (3) the intersection of any two soft sets in τ belongs to τ. The triplet (X, τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X. A soft set (M, E) over X is said to be a soft closed set in X, if its relative complement $(M, E)'$ belongs to τ.

Let (X, τ, E) be a soft topological space over X and (M, E) be a soft set over X. Then, the soft closure of (M, E), denoted by $\overline{(M, E)}$ is the intersection of all soft closed supersets of (M,E). Clearly $\overline{(M, E)}$ is the smallest soft closed set over X which contains (M, E). The soft interior of (M, E), denoted by $(M, E)^\circ$ is the union of all soft open subsets of (M, E). Clearly $(M, E)^\circ$ is the largest soft open set over X which is contained in (M, E).

A soft set (A, E) is a soft regular closed set if $\left[\left(\overline{A, E}\right)^\circ\right] = (A, E)$ and soft regular open if $\left[\overline{\left(A, E\right)}\right]^\circ = (A, E)$. A soft set (A, E) is a soft g-closed set if $\overline{(A, E)} \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X and A soft set (A, E) is a soft rg-closed set if $\overline{(A, E)} \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft regular open in X.

3. Soft Strongly *g*-closed Sets

Definition 3.1 A soft set (A, E) is a soft strongly g-closed set if $\left[\left(\overline{A, E}\right)^\circ\right] \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X.

Example 3.2 Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and $\tau = \left\{\tilde{\phi}, \tilde{X}, (A, E), (B, E), (C, E)\right\}$ be a soft topology defined on X, where (A, E), (B, E), (C, E) are soft sets over X, defined as follows: $A(e_1) = \phi$, $A(e_2) = \{a\}$, $B(e_1) = \{b\}$, $B(e_2) = \phi$, $C(e_1) = \{b\}$, $C(e_2) = \{a\}$. Then, (D, E), (F, E), (G, E) are the soft strongly g-closed where (D, E), (F, E), (G, E) are defined by $D(e_1) = \phi$, $D(e_2) = \{a, b\}$, $F(e_1) = \{a, b\}$, $F(e_2) = \{a\}$, $G(e_1) = \{b\}$, $G(e_2) = \{b\}$.

Theorem 3.3 Every soft closed set is a soft strongly g-closed set.

Proof. Let (A, E) be a soft closed set. Let $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X. Since (A, E) is soft closed, $\left[\left(\overline{A, E}\right)^\circ\right] \tilde{\subseteq} \overline{(A, E)} = (A, E) \tilde{\subseteq} (U, E)$ Therefore, (A, E) is soft strongly g-closed. □

Remark 3.4 The converse of the above theorem is not true in general. The following example supports our claim.

Example 3.5 In Example 3.2, (H, E) is soft strongly g-closed, but not soft closed where (H, E) is defined as follows: $H(e_1) = \{a\}$, $H(e_2) = \phi$.

Theorem 3.6 Every soft g-closed set is soft strongly g-closed.

Proof. Let $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X. Since (A, E) is soft g-closed, $\left[\overline{(A, E)^\circ}\right] \tilde{\subseteq} \overline{(A, E)} \tilde{\subseteq} (U, E)$. Therefore, (A, E) is soft strongly g-closed. □

Example 3.7 Let $X = (a, b\}$, $E = \{e_1, e_2\}$ and $\tau = \left\{\tilde{\phi}, \tilde{X}, (I, X), (J, E), (K, E), (L, E)\right\}$ be a soft topology defined on X, where (I, E), (J, E), (K, E), (L, E) are soft sets over X, defined as follows: $I(e_1) = \{a\}$, $I(e_2) = \phi$, $J(e_1) = \{a\}$, $J(e_2) = (a, b\}$, $K(e_1) = \{a\}$, $K(e_2) = \{a\}$, $L(e_1) = \{a\}$, $L(e_2) = \{b\}$. Then, (M, E) is soft strongly g-closed, but not soft g-closed where (M, E) is defined as follows: $M(e_1) = \phi$, $M(e_2) = \{b\}$.

Theorem 3.8 If a soft subset of a soft topological space X is both soft open and soft strongly g-closed, then it is soft closed.

Proof. Since (A, E) is soft open and (A, E) is soft strongly g-closed, $\left[\overline{(A, E)^\circ}\right] \tilde{\subseteq} (A, E)$. Since (A, E) is soft open, $\overline{(A, E)} = \left[\overline{(A, E)^\circ}\right] \tilde{\subseteq} (A, E)$. Therefore, (A, E) is soft closed. □

Corollary 3.9 If (A, E) is both soft open and soft strongly g-closed in X, then it is both soft regular open and soft regular closed in X.

Proof. Since (A, E) is soft open and soft strongly g-closed, (A, E) is soft closed by Theorem 3.8. Since (A,E) is soft open, $\left[\overline{(A, E)}\right]^\circ = [A, E]^\circ = (A, E)$. Therefore, (A, E) is soft regular open. Since (A, E) is soft closed, $(A, E) = \overline{(A, E)}$. Hence $(A, E) = \left[\overline{(A, E)^\circ}\right]$, since (A, E) is soft open. Thus (A, E) is soft regular closed.

Corollary 3.10 If (*A*, *E*) is both soft open and soft strongly *g*-closed in *X*, then it is soft *rg*-closed in *X*.

Theorem 3.11 A soft set (*A*, *E*) is soft strongly *g*-closed if $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ contains no non empty soft *g*-closed set.

Proof. Suppose that (*F*, *E*) is a soft *g*-closed subset of $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$. Then, $(F,E)\tilde{\subseteq}\overline{\left[(A,E)^{\circ}\right]}\tilde{\cap}(A,E)'$. Therefore, $(F,E)\tilde{\subseteq}(A,E)'$ implies that $(A,E)\tilde{\subseteq}(F,E)'$. Here (*F*, *E*)′ is soft *g*-open and (*A*, *E*) is soft strongly *g*-closed, we have $\overline{\left[(A,E)^{\circ}\right]}\tilde{\subseteq}(F,E)'$. Thus $(F,E)\tilde{\subseteq}\left[\overline{(A,E)^{\circ}}\right]'$. Hence $(F,E)\tilde{\subseteq}\left[\overline{(A,E)^{\circ}}\right]\tilde{\cap}\left[\overline{\left[(A,E)^{\circ}\right]}\right]'=\tilde{\phi}$. Therefore $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ contains no non empty soft g-closed set. □

Corollary 3.12 A soft strongly *g*-closed set (*A*, *E*) is soft regular closed if and only if $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ is soft *g*-closed and $(A,E)\tilde{\subseteq}\left[(A,E)^{\circ}\right]$.

Proof. Since (*A*, *E*) is soft regular closed, $\overline{\left[(A,E)^{\circ}\right]}=A,\overline{\left[(A,E)^{\circ}\right]}-(A,E)=\tilde{\phi}$ is soft g-closed.

Conversely, suppose that $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ is soft *g*-closed. Since (*A*, *E*) is soft strongly *g*-closed, $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ contains no non empty soft *g*-closed set by Theorem 3.11. Since $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ is itself soft *g*-closed, $\overline{\left[(A,E)^{\circ}\right]}=(A,E)$. Hence (*A*, *E*) is soft regular closed.

Theorem 3.13 A soft set (*A*, *E*) is soft strongly *g*-closed if $\overline{\left[(A,E)^{\circ}\right]}=(A,E)$. contains no non empty soft closed set.

Proof. Suppose that (*F*, *E*) is a soft closed subset of $\overline{\left[(A,E)^{\circ}\right]}=(A,E)$. Then, $(F,E)\tilde{\subseteq}\overline{\left[(A,E)^{\circ}\right]}\tilde{\cap}(A,E)'$. Therefore, $(F,E)\tilde{\subseteq}(A,E)'$ implies that $(A,E)\tilde{\subseteq}(F,E)'$. Here (*F*, *E*)′ is soft *g*-open and (*A*, *E*) is soft strongly *g*-closed, we have $\overline{\left[(A,E)^{\circ}\right]}\tilde{\subseteq}(F,E)'$. Thus $(F,E)\tilde{\subseteq}\left[\overline{\left[(A,E)^{\circ}\right]}\right]'$. Hence $(F,E)\tilde{\subseteq}\overline{\left[(A,E)^{\circ}\right]}\tilde{\cap}\left[\overline{\left[(A,E)^{\circ}\right]}\right]'=\tilde{\phi}$. Therefore $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ contains no non empty soft closed set.

Corollary 3.14 A soft strongly *g*-closed set (*A*, *E*) is soft regular closed if and only if $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ is soft closed and $(A,E)\tilde{\subseteq}\left[(A,E)^{\circ}\right]$.

Proof. Since (*A*, *E*) is soft regular closed, $\overline{\left[(A,E)^{\circ}\right]}=A,\overline{\left[(A,E)^{\circ}\right]}-(A,E)=\tilde{\phi}$ is soft closed.

Conversely, suppose that $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ is soft closed. Since (*A*, *E*) is soft strongly *g*-closed, $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ contains no non empty soft closed set by Theorem 3.13. Since $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ is itself soft closed, $\overline{\left[(A,E)^{\circ}\right]}-(A,E)$ Hence (*A*, *E*) is soft regular closed. □

Theorem 3.15 If (*A*, *E*) is soft strongly *g*-closed and $(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}\overline{\left[(A,E)^{\circ}\right]}$ then (*B*, *E*) is soft strongly *g*-closed.

Proof. Let $(B,E)\tilde{\subseteq}(G,E),(G,E)$ is soft open. Since $(A,E)\tilde{\subseteq}(B,E),(B,E)\tilde{\subseteq}(G,E),(A,E)\tilde{\subseteq}(G,E)$. Since (*A*, *E*) is soft strongly *g*-closed, $\overline{\left[(A,E)^{\circ}\right]}\tilde{\subseteq}(G,E)$. But $\overline{\left[(B,E)^{\circ}\right]}\tilde{\subseteq}\overline{\left[(A,E)^{\circ}\right]}$ implies $\overline{\left[(B,E)^{\circ}\right]}\tilde{\subseteq}(G,E)$. Therefore (*B*, *E*) is soft strongly *g*-closed.

Remark 3.16 If (*A*, *E*) and (*B*, *E*) are soft strongly *g*-closed then both $(A,E)\tilde{\cap}(B,E)$. and $(A,E)\tilde{\cup}(B,E)$ need not be soft strongly *g*-closed. The following examples support our claim.

Example 3.17 In Example 3.2, (*N*, *E*) and (*O*, *E*) are soft strongly *g*-closed in *X* where (*N*, *E*), (*O*, *E*) are soft sets over *X*, defined as follows: $N(e_1)=\{a,b\}$, $N(e_2)=\phi$, $O(e_1)=\{b\}$, $O(e_2)=\{b\}$. But $(N,E)\tilde{\cap}(O,E)=(P,E)$ is not soft strongly *g*-closed in *X* where (*P*, *E*) is defined as follows: $P(e_1)=\{b\}$, $P(e_2)=\phi$.

Example 3.18 Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ and $\tau=\left\{\tilde{\phi},\tilde{X},(Q,E),(R,E),(S,E)\right\}$ be a soft topology defined on *X*, where (*Q*, *E*), (*R*, *E*), (*S*, *E*) are soft sets over *X*, defined as follows: $Q(e_1)=\{a,b\}$, $Q(e_2)=\{b\}$, $R(e_1)=\{a\}$, $R(e_2)=\{a,b\}$, $S(e_1)=\{a\}$, $S(e_2)=\{b\}$. Then, (*T*, *E*) and (*U*, *E*) are soft strongly *g*-closed where (*T*, *E*) and (*U*, *E*) are defined as follows: $T(e_1)=\phi$, $T(e_2)=\{b\}$, $U(e_1)=\{a\}$, $U(e_2)=\phi$. Then, $(T,E)\tilde{\cup}(U,E)=(V,E)$ is not soft strongly *g*-closed in *X* where (*V*, *E*) is defined as follows: $V(e_1)=\{a\}$, $V(e_2)=\{b\}$.

4. Soft strongly *g*-open Sets

Definition 4.1 A soft set (*A*, *E*) is a soft strongly *g*-open set if its complement (*A*, *E*)′ is soft strongly *g*-closed in *X*.

Example 4.2 In Example 3.2, $(D, E)'$, $(F, E)'$, $(G, E)'$ are the soft strongly g-open where $(D, E)'$, $(F, E)'$, $(G, E)'$ are defined by $D'(e_1) = \{a, b\}$, $D'(e_2) = \phi$, $F'(e_1) = \phi$, $F'(e_2) = \{b\}$, $G'(e_1) = \{a\}$, $G'(e_2) = \{a\}$.

Theorem 4.3 A soft set (A, E) is a soft strongly g-open set if and only if $(F,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$ whenever $(F,E)\tilde{\subseteq}(A,E)$ and (F, E) is soft closed in X.

Proof. Necessity: Let $(F,E)\tilde{\subseteq}(A,E)$ and (F, E) is soft closed in X. Then, $(A,E)'\tilde{\subseteq}(F,E)'$ and $(F, E)'$ is soft open in X. Since $(A, E)'$ is soft strongly g-closed, $\left[\overline{\left((A,E)'\right)^{\circ}}\right]\tilde{\subseteq}(F,E)'$. Consequently,

$$(F,E)\tilde{\subseteq}\left\{\left[\overline{\left((A,E)'\right)^{\circ}}\right]\right\}'=\left\{\left[\overline{\left((A,E)'\right)^{\circ}}\right]'\right\}^{\circ}=\left[\overline{(A,E)}\right]^{\circ}.$$

Sufficiency: Let $(A,E)'\tilde{\subseteq}(U,E)$ and (U, E) is soft open in X. Then, $(U,E)'\tilde{\subseteq}(A,E)$ and $(U, E)'$ is soft closed in X. By our assumption, $(U,E)'\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$. This implies that $\left[\overline{\left((A,E)'\right)^{\circ}}\right]\tilde{\subseteq}(U,E)$. Hence $(A, E)'$ is soft strongly g-closed in X. Consequently, (A, E) is a soft strongly g-open set.

Theorem 4.4 Every soft open set is a soft strongly g-open set.

Proof. Let (A, E) be soft open set. Let $(F,E)\tilde{\subseteq}(A,E)$ and (F, E) is soft closed in X . Since (A, E) is soft open, $(F,E)\tilde{\subseteq}(A,E)=(A,E)^{\circ}\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$. Therefore, (A, E) is soft strongly g-open. ☐

Remark 4.5 The converse of the above theorem is not true in general. The following example supports our claim.

Example 4.6 In Example 3.2, $(H, E)'$ is soft strongly g-open, but not soft open where $(H, E)'$ is defined as follows: $H'(e_1) = \{b\}$, $H'(e_2) = \{a, b\}$.

Theorem 4.7 Every soft g-open set is soft strongly g-open.

Proof. Let (A, E) be soft g-open set in X. Let $(F,E)\tilde{\subseteq}(A,E)$ and (F, E) is soft closed in X. Since (A, E) is soft g-open, $(F,E)\tilde{\subseteq}(A,E)^{\circ}\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$. Therefore, (A, E) is soft strongly g-open. ☐

Example 4.8 In Example 3.7, $(M, E)'$ is soft strongly g-open, but not soft g-open where $(M, E)'$ is defined as follows: $M'(e_1) = \{a, b\}$, $M'(e_2) = \{a\}$.

Theorem 4.9 If a soft subset of a soft topological space X is both soft closed and soft strongly g-open, then it is soft open.

Proof. Since (A, E) is soft strongly g-open, $(A,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$. Since (A, E) is closed, $(A,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}=(A,E)^{\circ}$. Therefore, (A, E) is soft open.

Corollary 4.10 If (A, E) is both soft closed and soft strongly g-open in X, then it is both soft regular open and soft regular closed in X.

Proof. Since (A, E) is soft closed and soft strongly g-open, (A, E) is soft open by Theorem 4.9. Since (A, E) is soft closed, $\left[\overline{(A,E)}\right]^{\circ}=(A,E)^{\circ}=(A,E)$. Therefore, (A, E) is soft regular open. Since (A, E) is soft closed, $(A,E)=\overline{(A,E)}$. Hence $(A,E)=\left[\overline{(A,E)^{\circ}}\right]$, since (A, E) is soft open. Thus (A, E) is soft regular closed. ☐

Corollary 4.11 If (A, E) is both soft closed and soft strongly g-open in X, then it is soft rg-open in X.

Theorem 4.12 A soft set (A, E) is soft strongly g-open if $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ contains no non empty soft closed set.

Proof. Suppose that (F, E) is a soft closed subset of $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$. Then, $(F,E)\tilde{\subseteq}(A,E)\tilde{\cap}\left\{\left[\overline{(A,E)}\right]^{\circ}\right\}'$. Therefore, $(F,E)\tilde{\subseteq}(A,E)$ and (F, E) is soft closed. Since (A, E) is soft strongly g-open, we have $(F,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$. Also, $(F,E)\tilde{\subseteq}\left\{\left[\overline{(A,E)}\right]^{\circ}\right\}'$. Hence $(F,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}\tilde{\cap}\left\{\left[\overline{(A,E)}\right]^{\circ}\right\}'=\tilde{\phi}$. Therefore $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ no non empty soft closed set.

Corollary 4.13 A soft strongly g-open set (A, E) is soft regular open if and only if $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ is soft closed and $(A,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$.

Proof. Since (A, E) is soft regular open, $\left[\overline{(A,E)}\right]^{\circ}=(A,E),(A,E)-\left[\overline{(A,E)}\right]^{\circ}=\tilde{\phi}$ is soft closed.

Conversely suppose that $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ is soft closed. Since (A, E) is soft strongly g-open, $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ contains no non empty soft closed set by Theorem 4.12. Since $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ is itself soft closed, $\left[\overline{(A,E)}\right]^{\circ}=(A,E)$. Hence (A, E) is soft regular open.

Theorem 4.14 A soft set (A, E) is soft strongly g-open if $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ contains no non empty soft closed set.

Proof. Suppose that (F, E) is a soft closed subset of $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ Then, $(F,E)\tilde{\subseteq} A\tilde{\cap}\left\{\left[\overline{(A,E)}\right]^{\circ}\right\}'$. Therefore, $(F,E)\tilde{\subseteq}(A,E)$ and (F, E) is soft closed. Since (A, E) is soft strongly g-open, we have $(F,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$. Also, $(F,E)\tilde{\subseteq}\left\{\left[\overline{(A,E)}\right]^{\circ}\right\}'$. Hence $(F,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}\tilde{\cap}\left\{\left[\overline{(A,E)}\right]^{\circ}\right\}'=\tilde{\phi}$. Therefore, $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ no non empty soft closed set.

Corollary 4.15 A soft strongly g-open set (A, E) is soft regular open if and only if $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ is soft closed and $(A,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$.

Proof. Since (A, E) is soft regular open, $\left[\overline{(A,E)}\right]^{\circ}=(A,E)$, $\left[\overline{(A,E)}\right]^{\circ}=(A,E), (A,E)-\left[\overline{(A,E)}\right]^{\circ}=\tilde{\phi}$ is soft closed.

Conversely suppose that $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ is soft closed. Since (A, E) is soft strongly g-open, $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ contains no non empty soft closed set by Theorem 4.14. Since $(A,E)-\left[\overline{(A,E)}\right]^{\circ}$ is itself soft closed, $\left[\overline{(A,E)}\right]^{\circ}=(A,E)$. Hence (A, E) is soft regular open.

Theorem 4.16 If (A, E) is soft strongly g-open and $\left[\overline{(A,E)}\right]^{\circ}\tilde{\subseteq}(B,E)\tilde{\subseteq}(A,E)$ then (B, E) is soft strongly g-open.

Proof. Let $(F,E)\tilde{\subseteq}(B,E)$, (F, E) is soft closed. Since $(B,E)\tilde{\subseteq}(A,E)$, $(F,E)\tilde{\subseteq}(B,E)$, $(F,E)\tilde{\subseteq}(A,E)$. Since (A, E) is soft strongly g-open, $(F,E)\tilde{\subseteq}\left[\overline{(A,E)}\right]^{\circ}$. But $\left[\overline{(A,E)}\right]^{\circ}\tilde{\subseteq}\left[\overline{(B,E)}\right]^{\circ}$ implies $(F,E)\tilde{\subseteq}\left[\overline{(B,E)}\right]^{\circ}$. Therefore, (B, E) is soft strongly g-open. □

Remark 4.17 If (A, E) and (B, E) are soft strongly g-open then both $(A,E)\tilde{\cap}(B,E)$ and $(A,E)\tilde{\cup}(B,E)$ need not be soft strongly g-open. The following examples support our claim.

Example 4.18 In Example 3.2, $(N, E)'$ and $(O, E)'$ are soft strongly g-open in X where $(N, E)'$, $(O, E)'$ are soft sets over X, defined as follows: $N'(e_1) = \phi$, $N'(e_2) = \{a, b\}$, $O'(e_1) = \{a\}$, $O'(e_2) = \{a\}$. But $(N,E)'\tilde{\cup}(O,E)'=(P,E)'$ is not soft strongly g-open in X where $(P, E)'$ is defined as follows: $P'(e_1) = \{a\}$, $P'(e_2) = \{a, b\}$.

Example 4.19 In Example 3.18, $(T, E)'$ and $(U, E)'$ are soft strongly g-open where $(T, E)'$ and $(U, E)'$ are defined as follows: $T'(e_1) = \{a, b\}$, $T'(e_2) = \{a\}$, $U'(e_1) = \{b\}$, $U'(e_2) = \{a, b\}$. Then, $(T,E)'\tilde{\cap}(U,E)'=(V,E)'$ is not soft strongly g-open in X where $(V, E)'$ is defined as follows: $V'(e_1) = \{b\}$, $V'(e_2) = \{a\}$.

5. Conclusion

Thus, the notions of soft strongly g-closed sets and soft strongly g-open sets have been introduced and investigated. In future, the varieties of new continuous mappings and separation axioms based on these sets may be introduced and the future research may be undertaken in this direction.

6. References

1. Molodtsov D. Soft set theory first results. Comput Math Appl. 1999; 37(4–5):19–31.
2. Zadeh LA. Fuzzy sets. Inf Control. 1965; 8:338–53.
3. Atanassov K. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1986; 20(1):87–96.
4. Atanassov K. Operators over interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1994; 64(2):159–74.
5. Gorzalzany MB. A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy Sets and Systems. 1987; 21(1):1–17.
6. Pawlak Z. Rough sets. Int J Comput Sci. 1982; 11(5):341–56.
7. Maji PK, Biswas R, Roy R. An application of soft sets in a decision making problem. Comput Math Appl. 2002; 44(8–9):1077–83.
8. Maji PK, Biswas R, Roy R. Soft set theory. Comput Math Appl. 2003; 45(4–5):555–62.
9. Chen D. The parametrization reduction of soft sets and its applications. Comput Math Appl. 2005; 49(5–6):757–63.
10. Pie D, Miao D. From soft sets to information systems. Granular computing IEEE Inter Conference; 2005 25–27 Jul; 2:617–21.

11. Shabir M, Naz M. On soft topological spaces. Comput Math Appl. 2011; 61(7):1786–99.
12. Levine N. Generalized closed sets in topology. Rend Circ Mat Palermo. 1970; 19(2):89–96.
13. Kannan K. On Levine's generalized closed sets: a survey. Research J Appl Sci Eng and Tech. 2012; 4(11):1612–5.
14. Kannan K. Soft Generalized closed sets in soft topological spaces. Journal of Theoretical and Applied Information Technology. 2012; 37(1):17–21.
15. Yiiksel S, Tozlu N, Ergul ZG. On Soft generalized closed sets in soft topological spaces. Journal of Theoretical and Applied Information Technology. 2013; 55(2):273–9.
16. Kannan K, Rajalakshmi D. Soft semi star generalized closed sets. Malaysian Journal of Mathematical Sciences. 2015; 9(1):77–88.
17. Helen PM, Selvarani P, Vijayan V, Tharani P. Strongly g-Closed Sets and strongly g**-closed sets. Int Journal of Math Archives. 2012; 3(8):3108–12.