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# Performance Analysis of Variants of Differential Evolution on Multi-Objective Optimization Problems

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## Abstract

Differential Evolution (DE) algorithm is a stochastic search algorithm, applied to solve various optimization problems. Different DE variants such as rand/1/bin, best/1/bin, rand/2/bin, best/2/bin, etc. are existed in the literature and many comparative performance analyses among these DE variants in solving different single-objective optimization problems were already done by many researchers. But many real world applications are of the category of multi-objective optimization problems. Only minimal amount of research work has been found in the literature on the performance analysis of these DE variants on solving multi-objective kind of problems. In this paper, we analyze the performance of the Differential Evolution (DE) variants to solve Multi-objective Optimization Problems (MOP). We have chosen five multi-objective benchmark functions called ZDT test functions that are grouped by characteristics like convex, non-convex, non-uniform, discrete and low density pareto fronts. We used the DE variants of type DE/rand/1/*, DE/rand/2/*, DE/best/1/*, DE/best/2/* and DE/rand-to-best/1/*, where * represents the binomial/exponential crossover operation, to test the five ZDT functions. The performance analysis among these variants on multi-objective functions are performed based on the convergence and diversity nature of the solutions and analyzed using metrics called Convergence Metric ($C_m$) and Diversity Metric ($D_m$). The results show that the DE variants rand/1/bin and best/1/bin have the better performance in terms of Convergence and Diversity in the solution space in solving the above mentioned test functions. It is also possible to do the further analysis on these variants by applying them in parallel (i.e. more than one variant/algorithm is used to solve the problem) and by exchanging the information among them, to improve the solution.

**Keywords:** Convergence Metric, Differential Evolution, Diversity Metric, Multi-Objective Optimization Problems, Zitzler Deb Thiele (ZDT)

## 1. Introduction

Optimizing a problem involves finding the best possible values of the deciding factors of the problem. The fine-tuned values of these variables or factors, either maximize or minimize the solution of the given problem. Further, the problem may have either a single solution or multiple conflicting solutions. They are termed as Single-objective Optimization Problem (SOP) and Multi-objective Optimization Problem (MOP) respectively. Among many existing algorithms, Evolutionary Algorithms (EA) were found to be effective in finding near optimal solutions. Some of the evolutionary algorithms are Genetic Algorithms (GA), Evolution Strategies (ES), and Strength Pareto Evolutionary Algorithms (SPEA), Differential Evolution (DE). In this paper, we concentrate on Differential Evolution algorithms for solving MOP.

Nowadays, most of the real time optimization problems are of multi-objective type. Multi-objective problems can be seen in many fields like science, engineering, economics and so on. A MOP consists of more than one conflicting objective functions. In MOP, there doesn't exist any single solution that can simultaneously optimize all objective functions. Instead, it will have set of solutions that are optimal, which is called as Pareto Front. There exists two requirements of MOP: 1 to find out solutions

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that converges to the pareto optimal front, and 2 to find solution that will maintain diversity in population.

In this paper, the performance of different variants of DE is analyzed by applying them on five ZDT benchmark test functions. The analysis is done based on the convergence and diversity nature of solutions obtained by each variant. This nature is measured using Convergence Metric ($C_m$) and Diversity Metric ($D_m$).

## 2. Related Work

Collello et al[1], have provided an overview about the approaches in the Evolutionary Algorithms (EA). It consists of various EAs and other metaheuristics that can be used to solve MOP. These includes genetic algorithms, evolution strategies, particle swarm organization, cultural algorithms, differential evolution, ant colony, mathemetic algorithm and so on.

DE was proposed by Storn and Price[2] for single objective optimization problems. Over span of years, DE became successful and reputed optimization algorithm among researchers. Abbass et al[3,4] were the first to apply DE over MOP in algorithm called Pareto Differential Evolution (PDE). In this algorithm, new individuals are created by DE and only non-dominated individuals are kept for next generation.

Madavan[5] introduced Pareto Differential Evolution Approach (PDEA), which also creates new individuals like in PDE but in this approach DE combines parent population as well as new individuals and then ranks the obtained solutions using pareto based ranking as well as diversity ranking where the latter uses crowding distance metric of each individual.

Xue et al[6] introduced a new algorithm for MOP called Multi-Objective Differential Evolution (MODE), which initially found the fitness of individuals with the help of pareto based ranking and then analyzed diversity using crowding distance. The fitness value obtained was used to find out best individuals for next population. Later, Robic and Filipic[9] introduced an algorithm called Differential Evolution for Multi-objective Optimization (DEMO) which is explained clearly in section 4.2.

Mezura-Montes et al[10] have provided an overview about DE for solving MOP. DE algorithm and its variants are discussed in this paper. Also, the approaches of DE that can be used for faster convergence and maintaining diversity of optimal solutions is explained.

Jinil Persis et al[11] have addressed an application of multi-objective optimisation problem called Mobile Ad-hoc Network (MANET). MANET uses network performance measures such as delay, hop distance, load, cost and reliability. Ant based routing algorithm is used to solve MANET route optimisation problem.

Mahmoud et al[12] have addressed another application of multi-objective optimisation problem called land use planning based on sustainable development. The objectives of land use planning includes maximizing compactness, maximizing floor area ratio, maximizing compatibility, maximizing economic benefit and maximizing mix use. Non-dominated sorting genetic algorithm version two (NSGA-II) is used to solve this land use planning problem.

## 3. Multi-Objective Optimization Problems

Multi-objective optimization problems are the problem that has more than one conflicting objective function that has to be optimized simultaneously. The set of solutions obtained for MOP are called pareto optimal front. It consists of decision variables, which are the numerical quantities that solve the optimization problems. These objective functions are also subjected to certain constraints which could be either inequality constraints or equality constraints.

A general MOP includes a set of 'n' decision variables, a set of 'k' objective functions, and a set of 'p' inequality constraints or a set of 'q' equality constraints.

The optimization goal is to
Maximize/Minimize

$Y = f(x) = (f_1(x), f_2(x)\ldots f_k(x))$

Subject to
Inequality Constraints

$e_i(x) \leq 0$, where $i = 1, 2\ldots p$

Or Equality Constraints

$g_j(x) = 0$, where $j = 1, 2\ldots q$

Where $x = (x_1, x_2\ldots x_n)$, Y denotes the objective vector and x denotes the decision vector.

# 4. Test Functions of MOP

Deb[7] in his experiments observed that the characteristics of pareto optimal front that prevent any evolutionary algorithm from finding diverse pareto optimal front are convexity, non-convexity, non-uniformity and discreteness. Considering these issues Zitzler, Deb, Thiele[8] developed five test functions described in the sections 4.1 to 4.5. These functions are termed as ZDT functions. These ZDT functions later became broad and popular set of benchmark test functions for analyzing multi-objective pareto optimization problems. Each ZDT function includes two objective functions, which is most common for pareto optimization, especially in engineering field.

Each ZDT function is the mathematical formulation of the characteristics like convexity, non-convexity, discreteness, non-uniformity, local pareto fronts and so on.

## 4.1 ZDT1

Decision space : $x \in [0, 1]^{30}$

Objective Function : $f_1(x) = f_2(x) = g(x)(1 - \sqrt{x_1/g(x)})$

$$g(x) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i$$

Optimal Solution : $0 \le x_1 \le 1$, $x_i = 0$ where $i = 2\ldots 30$

Characteristics : Convex Pareto Front

## 4.2 ZDT2

Decision space : $x \in [0,1]^{30}$

Objective Function : $f_1(x) = x_1$

: $f_2(x) = g(x)(1-(x_1/(g(x))^2))$

$$g(x) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i$$

Optimal Solution : $0 \le x_1 \le 1$, $x_i = 0$ where $i = 2\ldots 30$

Characteristics : Concave Pareto Front

## 4.3 ZDT3

Decision space : $x \in [0,1]^{30}$

Objective Function : $f_1(x)=x_1$

$$f_2(x) = g(x)\left(1 - \sqrt{x_1/g(x)} - \frac{x_1}{g(x)}\sin(10\pi x_1)\right)$$

$$g(x) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i$$

Optimal Solution : $0 \le x_1 \le 1$, $x_i = 0$ where $i=2\ldots 30$

Characteristics : Discontinuous Pareto Front

## 4.4 ZDT4

Decision space : $x \in [0,1] \times [-5,5]^9$

Objective Function : $f_1(x) = x_1$

$f_2(x) = g(x)(1-(x_1/(g(x))^2))$

$$g(x) = 1 + 10(n-1) + \sum_{i=2}^{n}(x_j^2 - 10\cos(4\pi x_j))$$

Optimal Solution : $0 \le x_1 \le 1$, $x_j = 0$ where $j = 2\ldots 10$

Characteristics : Many Local Pareto Front

## 4.5 ZDT6

Decision space : $x \in [0,1]^{30}$

Objective Function : $f_1(x) = 1 - \exp^{-4x_1}\sin(6\pi x_1)^6$

$f_2(x) = g(x)(1-(f_1(x)/(g(x))^2)$

$$g(x) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_j$$

Optimal Solution : $0 \le x_1 \le 1$, $x_j = 0$ where $j = 2\ldots 10$

Characteristics : Pareto Front having low density solutions near to it

# 5. Differential Evolution Algorithm

DE is one of the evolutionary algorithm used to solve many optimization problems. DE is a simple and powerful population based algorithm that uses real valued parameters. The advantages of DE include its ease of use, speed, simple structure and robustness. The three operations involved in the algorithm for optimizing the given objective functions are crossover, mutation and selection .The three strategy parameters used in DE are the size of the population (NP), Scaling Factor (F) and Crossover Rate (CR). The Scaling Factor (F) deals with population diversity. Smaller value of F leads to the premature convergence and hence leads to loss of diversity. The Crossover Rate (CR) controls the number of components inherited from the mutant vector. It influences the mutation probability and convergence speed. There exists many variants of DE that are discussed in the section 5.1.

## 5.1 De Variants

Price and Storn[1], initially introduced the DE algorithm with single variant for solving SOP. Later few more strategies of DE were suggested by them which are also called as DE variants. These variants vary based on the way the vectors are selected for computing mutation process, number of pairs of vectors to find the difference vector and the type of crossover used. The different variants and their mutation strategies are given in the Table 1.

The general convention of the DE variant can be denoted as DE/p/q/r. Here, the DE denotes the Differential Evolution Algorithm, 'p' represents the method of selecting the vectors for the mutation process (the vector could be the best vector of the current generation or randomly selected vector or a vector between the best and random vector), 'q' denotes the number of pairs of vector(s) to be selected to compute the difference vector and 'r' denotes the type of crossover to be performed which could be either bin or exp (where bin denotes for Binomial Crossover and exp denotes Exponential Crossover).

## 5.2 De for Mop

In this paper, the DEMO algorithm proposed by Robic and Filipic[9] is used to analyze the performance efficacy of the DE variants empirically on the previously mentioned ZDT functions. The DEMO algorithm uses the concept of *dominance*. That is, if the newly generated vector dominates its parent vector, then the new vector replaces the parent else, the new vector is discarded. The new vector is added into the population if both (new vector and parent vector) are non-dominated solutions.This may increase the size of the population. Therefore, non-dominated sorting and crowding distance measures are used to truncate the population. The DEMO algorithm is given in Figure 1 and the corresponding candidate creation using the variant DE/rand/1/bin is given in Figure 2.

1. Generate random initial population P.
   2. While stopping criterion not met, do:
      2.1. For each individual $X_j$ $(j = 1 \ldots NP)$ where NP is size of population
         Repeat
         (a) Create child$C$ from parent $X_j$.
         (b) Evaluate the child using the objective functions.
         (c) If the child dominates the parent, the child replaces the parent.
         Else, the candidate is discarded. If both are non-dominated, the candidate is added in the population.
      2.2. If the population has more than NP individuals, truncate it.
      2.3. Randomly enumerate the individuals in *P*.

**Figure 1.** Demo algorithm.

**Table 1.** Differential Evolution Variants Mutation Strategy

| Sl. No. | Variants | Mutation Strategy |
|---|---|---|
| 1 | DE/rand/1 | $V_{i,G} = X_{p_1{}^i,G} + F.(X_{p_2{}^i,G} - X_{p_3{}^i,G})$ |
| 2 | DE/rand/2 | $V_{i,G} = X_{p_1{}^i,G} + F.(X_{p_2{}^i,G} - X_{p_3{}^i,G} + X_{p_4{}^i,G} - X_{p_5{}^i,G})$ |
| 3 | DE/best/1 | $V_{i,G} = X_{best,G} + F.(X_{p_1{}^i,G} - X_{p_2{}^i,G})$ |
| 4 | DE/best/2 | $V_{i,G} = X_{best,G} + F.(X_{p_1{}^i,G} - X_{p_2{}^i,G} + X_{p_3{}^i,G} - X_{p_4{}^i,G})$ |
| 5 | DE/rand-to-best/1 | $V_{i,G} = X_{p_1{}^i,G} + K.\left(X_{best,G} - X_{p_3{}^i,G}\right) + F.(X_{p_1{}^i,G} - X_{p_2{}^i,G})$ |

*Input:* Parent *Pi*

1. Randomly select three individuals $X_{j1}, X_{j2}, X_{j3}$ from *P*, where $j \neq j_1 \neq j_2 \neq j_3$.
2. Calculate child C as C = $C = X_{j1} + F(X_{j2} - X_{j3})$ where *F* is a scaling factor.
3. Evaluate the child by crossover with the parent using crossover rate CR

*Output:* Child*C*

**Figure 2.** Candidate creation using DE/rand/1/bin.

## 6. Performance Metrics

The metrics used in this paper to analyze the performance of the DE variants are based on the convergence and diversity nature of the population. It should be noted that the DE variants having less convergence value and less diversity value are considered to be better performing variants. The Convergence metric and Diversity metric are explained in the Sections 6.1 and 6.2 respectively.

### 6.1 Convergence Metrics

The convergence metric $C_m$ is a measure that calculates the distance between the obtained non-dominated solutions B and the setof Pareto-optimal solutions. It tells how faster the obtained solutions converges to pareto front.

$$C_m = \left(\sum_{i=1}^{|Q|} d_i\right) / |B|$$

where $d_1$ is the distance between the solution i ∈ B and the nearest member of pareto optimal solution.

### 6.2 Diversity Metrics

The second metric is the Diversity metric $D_m$. This metric measures how diverse the obtained non-dominated solutions are.

$$D_m = \frac{d_f + d_l + \sum_{i=1}^{|Q|-1} d_i - \bar{d}}{d_f + d_l + (|Q|-1)\bar{d}}$$

where $d_i$ is the distance between consecutive solutions in the non-dominated solutions Q, and d is the average of all these distances.

The parameters $d_f$ and $d_l$ represent the distances between the extreme solutions of the Pareto front and the boundary solutions of the obtained solutions Q.

## 7 Experimental Design

The empirical analysis was performed with the following parameter values.
Size of the population, NP : 50
Number of runs : 30
Maximum number of generations, MAXGEN : 2000
Crossover Rate ( $CR_{min}$ , $CR_{max}$ ) : (0.3, 0.9)
Scaling Factor, F : 0.5

## 8. Results and Disscussion

The Convergence value $C_m$ of each variant tested on each ZDT benchmark test functions are given in the Table 2. The results shows that based on $C_m$, that is, the best performing variants in the order of lowest convergence metric are: 1. DE/rand/1/bin, DE/best/1/bin and DE/rand/2/bin for ZDT1, 2. DE/best/1/bin, DE/rand/1/bin and DE/rand/2/bin for ZDT2, 3. DE/rand/1/bin, DE/best/1/bin and DE/rand/2/bin for ZDT3, 4. DE/best/1/bin and DE/rand/1/bin for ZDT4, 5. DE/rand/1/bin and DE/best/1/bin for ZDT6. The variants DE/rand/1/bin and DE/best/1/bin perform better irrespective of specific ZDT function. The variants DE/best/2/bin and DE/rand-to-best/1/exp had higher $C_m$ value for ZDT1, ZDT2 and ZDT3 and DE/rand/2/bin and DE/best/2/exp had higher $C_m$ value for ZDT4 and ZDT6.

The Diversity value $D_m$ of each variant applied on each ZDT benchmark test functions are given in the Table 3. The result shows that on the basis of $D_m$, the variants performed best in the order of lowest diversity metric are: 1. DE/rand/1/bin and DE/best/1/bin for ZDT1, 2. DE/rand/1/bin and DE/best/1/bin for ZDT2, 3 DE/best/1/bin and DE/rand/1/bin for ZDT3, 4. DE/best/1/bin, DE/rand/1/bin and DE/best/2/bin for ZDT4, 5. DE/rand/1/bin, DE/best/1/bin and DE/best/2/bin for ZDT6. Again, the variants DE/rand/1/bin and DE/best/1/bin perform better irrespective of specific ZDT function. The variants DE/best/1/exp, DE/rand/2/exp and DE/best/2/exp had higher $D_m$ value for ZDT1, ZDT2 and ZDT3 and DE/rand/1/exp and DE/best/2/exp had higher $D_m$ value for ZDT4 and ZDT6.

## 9. Conclusion

In this paper, the performance efficacy of different variants of Differential Evolution algorithm are analyzed empirically based on their convergence and diversity nature. An empirical comparison of ten DE variants to solve ZDT multi-objective optimization problems was done. The performance of DE variants to solve MOP are analyzed by checking whether it fulfills the two goals of MOP, i. e., convergence to the pareto front and uniform diversity of the obtained solutions. These two goals are determined using two metrics called Convergence Metric and Diversity Metric. The results shows that the best performing variants are rand/1/bin, best/1/bin which had faster convergence to the pareto front and also have uniform spread of solutions along the front.

**Table 2.** Convergence Metric value for each ZDT Function. The lowest is made bold

| Variant | ZDT1 | ZDT2 | ZDT3 | ZDT4 | ZDT6 |
|---|---|---|---|---|---|
| *DE/rand/1/bin* | **2.7524E-317** | 1.8245E-317 | **1.6515E-317** | 3.0538E-317 | **1.8134E-317** |
| *DE/rand/1/exp* | 2.0088E-013 | 2.1088E-013 | 2.0288E-013 | 5.4645E-086 | 5.4835E-086 |
| *DE/rand/2/bin* | 1.2637E-306 | 1.2847E-306 | 1.2637E-306 | 7.3759E+097 | 7.5659E+097 |
| *DE/rand/2/exp* | 1.8305E-076 | 1.8415E-076 | 1.8375E-076 | 5.4885E-086 | 5.4945E-086 |
| *DE/best/1/bin* | 2.7635E-317 | **1.8134E-317** | 1.6626E-317 | **3.0438E-317** | 1.8245E-317 |
| *DE/best/1/exp* | 1.3860E+093 | 1.3871E+093 | 1.3860E+093 | 1.0796E+021 | 1.0786E+021 |
| *DE/best/2/bin* | 1.0253E+200 | 1.0266E+200 | 1.0300E+200 | 4.8663E+011 | 4.8713E+011 |
| *DE/best/2/exp* | 1.9889E+045 | 1.9888E+045 | 1.9866E+045 | 1.3867E+107 | 1.3877E+107 |
| *DE/rand-to-best/1/bin* | 1.3658E+074 | 1.3668E+074 | 1.3688E+074 | 6.7767E+073 | 6.7778E+073 |
| DE/rand-to-best/1/exp | 2.5874E+161 | 2.5894E+161 | 2.5869E+161 | 6.1293E+030 | 6.1298E+030 |

**Table 3.** Diversity Metric value for each ZDT Function. The lowest is made bold

| Variant | ZDT1 | ZDT2 | ZDT3 | ZDT4 | ZDT6 |
|---|---|---|---|---|---|
| *DE/rand/1/bin* | **1.9753E-317** | **2.6229E-317** | 3.7996E-317 | 2.9578E-317 | **2.6553E-317** |
| *DE/rand/1/exp* | 9.1154E+227 | 9.8154E+227 | 9.9254E+227 | 6.9389E+252 | 6.6589E+252 |
| *DE/rand/2/bin* | 5.7777E+199 | 6.7897E+199 | 7.7867E+199 | 1.3658E+074 | 1.5658E+074 |
| *DE/rand/2/exp* | 1.7706E+248 | 1.7726E+248 | 1.7709E+248 | 2.6425E+092 | 2.6665E+092 |
| *DE/best/1/bin* | 1.9864E-317 | 2.6340E-317 | **3.7885E-317** | **2.9467E-317** | 2.6664E-317 |
| *DE/best/1/exp* | 2.6326E+276 | 2.6315E+276 | 2.6318E+276 | 7.6222E+218 | 7.6892E+218 |
| *DE/best/2/bin* | 5.0140E+180 | 5.0150E+180 | 5.0177E+180 | 1.1243E-307 | 1.2243E-307 |
| *DE/best/2/exp* | 2.5060E+262 | 2.5040E+262 | 2.5120E+262 | 1.3800E+228 | 1.3780E+228 |
| *DE/rand-to-best/1/bin* | 8.9473E+073 | 8.9673E+073 | 8.9493E+073 | 1.8113E+045 | 1.9913E+045 |
| *DE/rand-to-best/1/exp* | 1.1616E-012 | 2.1716E-012 | 3.1876E-012 | 1.3981E-152 | 1.6781E-152 |

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