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Iterative Linear Quadratic Regulator (ILQR) Controller for Trolley Position Control of Quanser 3DOF Crane

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Abstract

In this paper, we have investigated the performance of Iterative Linear Quadratic Regulator (ILQR) on trolley position of 3DOF crane. In ILQR, we select optimum parameters Q and R automatically instead of hit and trial method. Algorithm chooses the parameters Q and R which results in minimum trolley's settling time of the jib system. A number of simulations have carried out using Matlab/Simulink. The results show that the optimized LQR results reduce settling time of trolley along with smaller overshoot with less rise time.

Keywords: Iterative Linear Quadratic Regulator (ILQR), Proportional Integral Derivative (PID), Three Degree of Freedom (3DOF)

1. Introduction

3DOF cranes are often used in a number of industries like construction, textile and sugar mills. It is often used for moving heavy objects from one location to another location. In the movement of heavy objects there is often an issue of payload swing. This swinging not only hinders the smooth motion of trolley of crane but in extreme cases destroys the whole mechanical structure. The electronic control of crane often needs a reasonable input signal of actuator to behave properly. The classical techniques are easy to implement for controlling of crane. The problem with classical control techniques is that it needs proper tuning for reasonable performance. It is still not possible to remove steady state error completely or make system totally stable with small effort. The fuzzy logic makes the implementation of controller easy. It needs input and output information of system irrespective of the system dynamics. It is a very simple method for controlling the system. The idea of fuzzy logic has extended to adaptive fuzzy for the better vibration control.

The LQR is a very effective approach for controlling the crane[8,9]. Major issue with this approach is that it needs complex computation for calculation of gains. The performance of fuzzy controller is better than the LQR controller for vibration control[10]. The neural network technique is very helpful for vibration control because it involves learning capability[11-17]. The individual limitations or draw backs of neural networks and fuzzy logic are effectively solved by using them together in the form of neuro-fuzzy[18-21].

In all of the above techniques, main objective was on designing a smooth trajectory for trolley having minimum payload vibrations. Overshoot and damping of oscillations are the two performance measures that are considered in controller design. The controller will be more efficient if more performance measures are involved. This needs more power, logic and computation. In this paper, we involve only one performance measure which is settling time of the trolley position.

The minimization of this performance measure i.e. settling time, converges the other parameters, like overshoot

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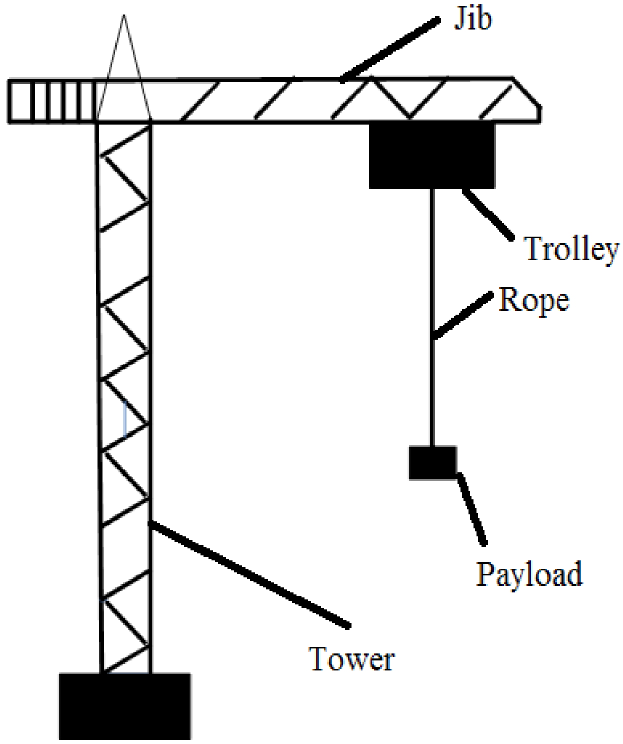

Figure 1. The schematic of 3DOF crane.

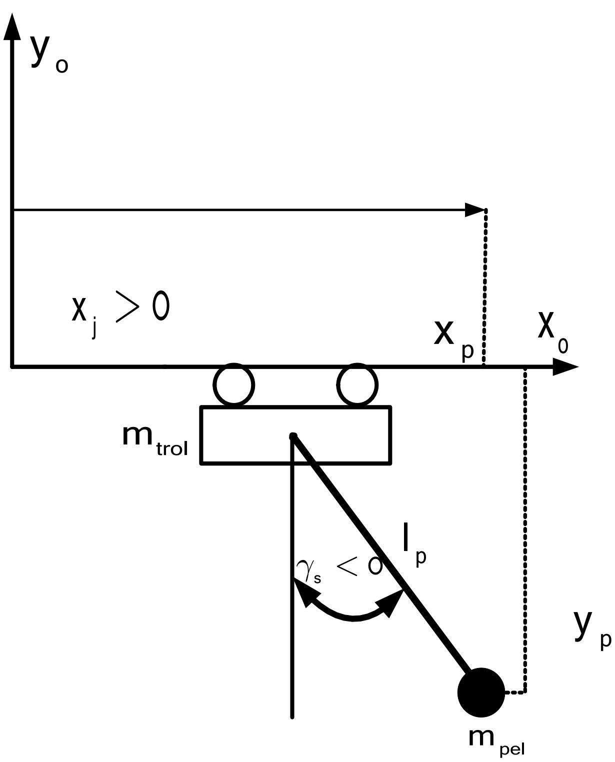

Figure 2. The free diagram of the jib system.

and rise time, to a very low value. We have developed and analyzed the iterative linear quadratic regulator for minimization of settling time of trolley position. The ILQR is actually optimization of LQR in recursive way. In which an algorithm takes the range of input parameters Q and R and then optimize the LQR automatically. Figure 1 shows the general schematic of 3DOF crane. The brief introduction of the topic is presented in section 1. In section 2, modeling of the system is discussed. In section 3 the proposed algorithm is explained. The simulation results are provided in section 4. Finally, in section 5, conclusion of the paper is given.

2. Modeling of the Jib System

The Quanser 3DOF crane prototype is an advance hardware in the loop for validation of controllers performance. The tower, payload and jib system are three main parts of the 3DOF crane. The jib trolley, jib on tower and payload involve a linear to and fro, clockwise/anti-clockwise and up/down movements respectively. The modeling of jib system is very similar to modeling of the inverted pendulum as is shown in the Figure 2. The linearization of nonlinear system has done through Lagrange method[22, 23].

The nonlinear system after linearization is represented in state space as is given below.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{m_{pel} r_{pul}{}^{2} g_{gr}}{m_{trol} r_{pul}{}^{2} + j_{\psi} k_{jg}{}^{2}} & 0 & 0 \\ 0 & -\dfrac{g\left(m_{trol} r_{pul}{}^{2} + m_{pel} r_{pul}{}^{2} + j_{\psi} k_{jg}{}^{2}\right)}{(m_{trol} r_{pul}{}^{2} + j_{\psi} k_{jg}{}^{2})\mathrm{I}_{pa}} & 0 & 0 \end{bmatrix} \tag{1}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{r_{pul} \eta_{jg} k_{jg} \eta_{jm} k_{jt}}{m_{trol} r_{pul}{}^{2} + j_{\psi} k_{jg}{}^{2}} \\ \dfrac{r_{pul} \eta_{jg} k_{jg} \eta_{jm} k_{t}}{(m_{trol} r_{pul}{}^{2} + j_{\psi} k_{jg}{}^{2}) I_{pa}} \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4}$$

Table 1. Nomenclature

Sl. No.	Symbol	Description	Sl. No.	Symbol	Description
1	K_t	Torque constant of Jib motor	8	I_{pa}	Perpendicular distance of Payload from Jib arm
2	η_{jm}	Efficiency of Jib motor	9	g_{gr}	Acceleration constant of gravity
3	k_{jg}	Gear ratio of Jib motor	10	m_{pel}	Mass of Payload
4	η_{jg}	Gearbox efficiency of Jib motor	11	A	System state matrix
5	r_{pul}	Radius of trolley pulley from pivot to end of tooth	12	B	Input matrix
6	m_{trol}	Mass of the trolley	13	C	Output matrix
7	J_{ψ}	Equivalent moment of inertia of Jib motor	14	D	Direct transmission

The variable in the state space are provided in Table 1 of Appendix. The interested reader can study the detail of Quanser 3DOF crane on its user manual[24].

3. Iterative Linear Quadratic Regulator (ILQR)

In the designing of ILQR, there is need of knowledge about LQR. The brief introduction of LQR is discussed. The reader can review detail in[25-30]. The two important conditions for designing LQR for a system is that it must be stable for infinite time and it should be controllable. The controllability matrix is given below

$$C_{cnt} = \begin{bmatrix} B & AB & A^2B\ldots \end{bmatrix} \tag{5}$$

In LQR, we often optimize the controller by minimizing the cost function J_{cntr} through using parameters Q and R. The formula of the cost function J_{cntr} is given below:

$$J_{cntr} = \int_0^\infty (x_{st}^T Q x_{st} + u_{inp}^T R u_{inp})dt \tag{6}$$

The u_{inp} represents control input to plant. The x_{st} represents the system states. Q and R are the weights on the basis of which the cost function minimizes the cost of control signal. The need of cost function is obvious, it is because of the reason that the parameter Q relates to output and parameter R relates to input. So, if there is any increase in two parameters to a certain limit, in order to get the optimum input or output, then there could be limitations in the hardware. In other words, just for the sake of optimal output, we can't afford the heavy input energy expenditure and vice versa. The parameters Q and R must be symmetric positive definite. After selection of appropriate cost function, next step is to find the control gain by the formula:

$$K_{cntr} = R^{-1}B^T P_{in} \tag{7}$$

Here, P_{in} is a performance index. In the context to performance index P_{in}, all states of the system must be observable. The matrix P_{in} can be found through the famous Algebraic Riccati equation as is represented below

$$A^T P_{in} + P_{in}A + Q = P_{in}BR^{-1}B^T P_{in} \tag{8}$$

Here, A, B represents the system matrices. After finding the controller gain K_{cntr}, the final control law is:

$$u_{ip} = -K_{cntr} x_{st} \tag{9}$$

The jib system fulfills the two necessary conditions that are: system is controllable or controllability matrix is full rank and system is stable for infinite time. So, LQR controller is applicable to the jib system of crane. Figure 3 shows the flow chart of ILQR algorithm, here, we just input the range of input parameters Q and R. The algorithm automatically chooses the parameters Q and R that find out optimum results. In ILQR, we start with the initial values of LQR input parameters Q and R as follows:

$$R = 0.1$$

$$Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.02 \end{bmatrix} \tag{10}$$

We have defined the range of total iterations to 100. At each iteration, the calculated value of settling time of the trolley, after implementation of controller on jib system, are stored in array. Then, in each iteration the value of LQR parameters increment as follows:

$$R = R + 0.01$$
$$Q = Q + 0.05 * I_{5\times5} \tag{11}$$

Here, $I_{5\times5}$ presents an identity matrix with five rows and five columns. After defined number of iterations, the value of minimum settling time and its location from array is achieved. So, with the help of location information, we trace out the LQR parameters Q and R. In our case, 75th iteration, our algorithm searches parameters Q and R which results optimum performance. The LQR parameters in 75th iteration are:

$$R = 0.750$$

$$Q = \begin{bmatrix} 0.475 & 0 & 0 & 0 & 0 \\ 0 & 0.475 & 0 & 0 & 0 \\ 0 & 0 & 0.395 & 0 & 0 \\ 0 & 0 & 0 & 0.475 & 0 \\ 0 & 0 & 0 & 0 & 0.395 \end{bmatrix} \tag{12}$$

Now we can find the gain using these optimal parameters and get the response of the system. Figure 4 shows the Simulink model of jib controller. Here, the jib control system has gains which are calculated by LQR. The controller is implemented by PID. If we use LQR on actual plant then it results PD gains, so, for development of PID controller, one row in last and one column in right of matrix A is added with all entries being zero except $A(5,1)$ and $A(5,5)$ which are 1. The plant system involves state space of the system. The plant system also involves actuator dynamics block, which is used to limit the current input to the plant. The observer involves low pass and high pass filters. These help to estimate all states by taking partial states. The partial-state and full-state switches decides either we are interested to involve only partial states namely position and rate of position or full states namely position, swing angle, rate of position and rate of swing angle respectively.

4. Simulation Results

In this research paper we have minimized the settling time of plant using Iterative Linear Quadratic Regulator (ILQR) algorithm. In ILQR algorithm, a range of input parameters Q and R are defined. Then ILQR automatically searches for the optimum value of settling time from set of data. The purpose of selecting minimum

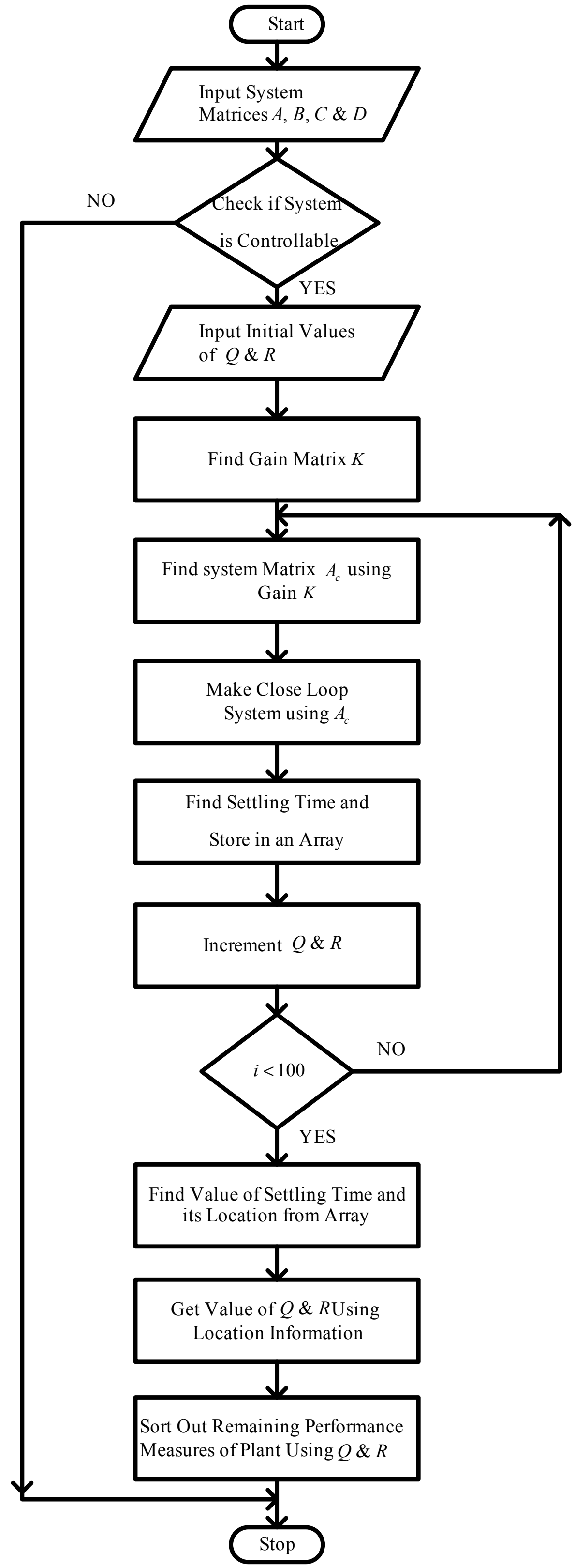

Figure 3. Flow chart of iterative linear quadratic regulator (ILQR).

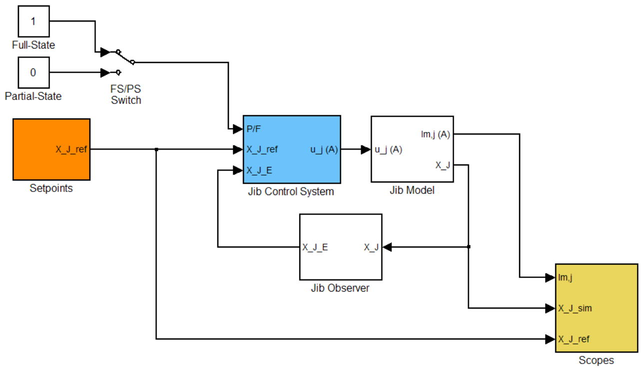

Figure 4. Simulink model of the jib system.

settling time is that it shows at which duration the system settles. Thus, providing sufficient knowledge of system's behavior. The Figure 5 and Figure 6 show the open loop response of the trolley position and payload swing angle of jib system of 3 DOF crane. It's observed that system is totally unstable. That is because of the reason that the trolley is not following any reference tracking. Also, the payload oscillates, along jib axis, continuously and with heavy amplitude. The amplitudes of swing angle are more than 64 degrees in counter clock wise direction and more than 137 degrees in clockwise direction respectively. This large value of payload oscillations and instability of trolley, results in uneven movement of the trolley movement. That is not favorable in smooth operation of crane.

The ILQR algorithm helps in finding suitable gains for smooth operation of crane. Figure 7 shows the performance of ILQR on trolley position of 3DOF crane. This implies that trolley overshoot converges from infinite amplitude to finite amplitude of value 28.3677. The trolley rise time is 0.9945. The minimum settling time that is obtained through ILQR algorithm is 4.9008. Figure 8 shows the performance of ILQR on payload swing angle. This implies that rise time of payload load swing is 0.0023. The settling time of payload swing is 4.3205. The amplitude of payload swing reduces from a value of 137.6 degrees to 9.065 degrees in clock wise direction. The amplitude of payload swing reduces from a value of 64.92 to 9.065 degrees in counter clockwise direction. Hence, minimum value of trolley settling time to ensure the smooth operation of crane is found.

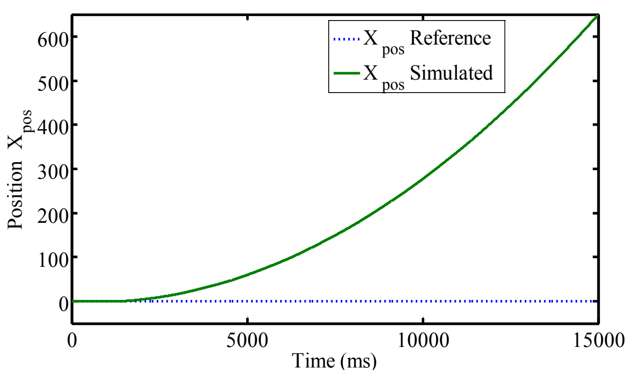

Figure 5. The open loop response of the trolley position of the jib system.

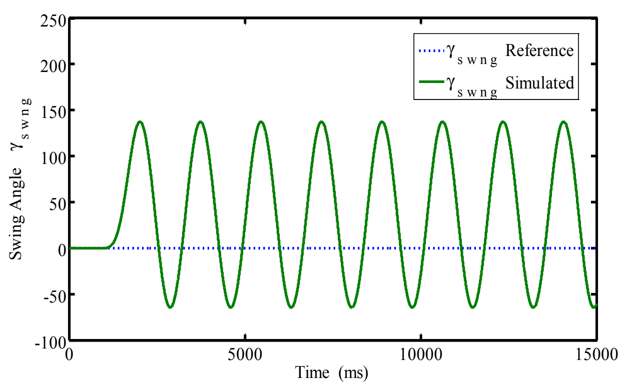

Figure 6. The open loop response of the payload vibration.

5. Conclusion

In this research work, we have investigated the Iterative Linear Quadratic Regulator (ILQR) controller in finding the minimum value of the settling time of the trolley position for the jib system of 3DOF crane. The ILQR is a very effective algorithm for designing optimum LQR controller. The simulations and experimental results show that the ILQR has brought the unstable trolley movement to stable motion along with the reduction of payload swing

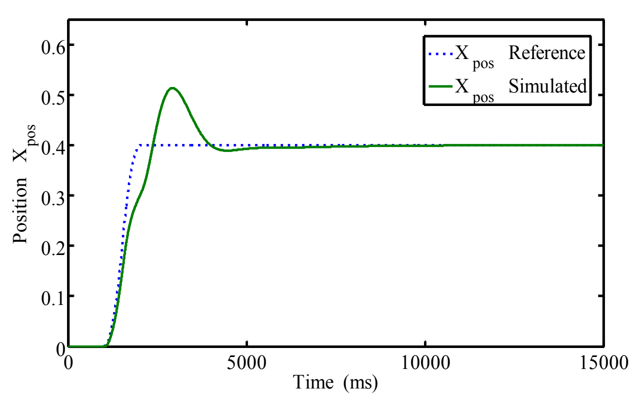

Figure 7. The performance of linear quadratic regulator (ILQR) on trolley position.

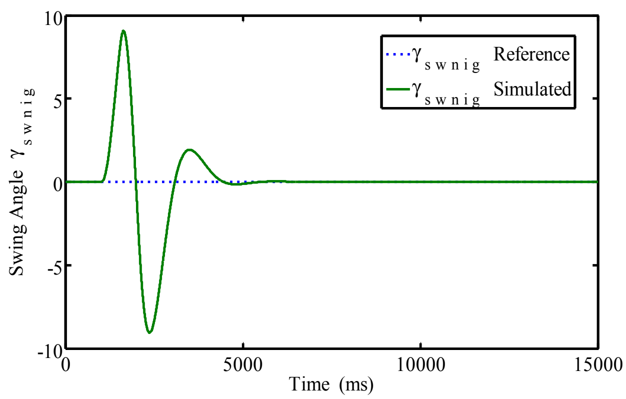

Figure 8. The performance of linear quadratic regulator (ILQR) on payload swing angle.

angle. The ILQR helps us in finding the optimum LQR by selecting the appropriate LQR parameters Q and R automatically. In our future work, we will optimize the LQR controller by involving more performance measures like rise time, overshoot along with settling time.

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