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Length Scales Analysis of Wave Scattering from Rough Surfaces

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Abstract

The aim is to study surface characteristics through scattered wave intensity in frame work of the Kirchhoff wave theory. There are three length scales which scaling behavior between them play role in rough surface scattering, wave-length (λ) of the incident wave, the roughness (σ) and the correlation length (ξ) of the surface. In this work we show the effective role of the correlation length in surface scattering. Up to now, some of the reports for wave scattering from rough surfaces are based on the product of the wave number and surface roughness $k\sigma$ without consideration to correlation length. For $\lambda \gg \xi$, correlation length has no effects on the properties of the scattered wave and $k\sigma$ is the appropriate parameter to study the rough surfaces scattering problem. But, if ξ and λ are in comparable range, scattered wave depends on how $k\sigma$ is chosen by k or σ. In this case, $k\sigma$ is not suitable to present wave scattering from rough surface. In other word, for a constant ka, scattered wave could depend on any of the parameters k or σ. To justify our statements we compare our theoretical results with experimental data for the intensity profile obtained from a self-affine rough surface. We show that by changing the parameters ξ, λ, σ, and the Hurst exponent, the intensity profiles obtained by theory and numerical estimation overlap when λ is much longer than ξ. But interestingly, the profiles start to diverge when ξ tends to λ. This provides a good understanding of the role of the characteristics of the surface on the profile of scattered intensity.

Keywords: Correlation Length, Kirchhoff Theory, Roughness, Wave Scattering

1. Introduction

Roughness effects on scattering intensity have been investigated by many researchers for the past few decades. Kirchhoff theory is one of the most used methods to study wave scattering problem[1-5]. It is suitable for rough surface with roughness is same order and smaller than wavelength, which is based on electromagnetic principles. Also, the surfaces with slight roughness are approximated by Rayleigh-Rice theory, which is a perturbed boundary condition method[6]. The scattered intensity depends on the properties of the rough surface and the incident field. Evaluating the intensity from a rough surface with known properties is called the direct problem[7-9]. The opposite question of the direct problem is called the inverse problem which is determining the properties of a surface by using the information embedded in measured scattered intensity[10-15].

Some of the scattering studies are based on variations in dimensionless $k\sigma$ parameter; where k and σ are two length scale in the system[3,16-18]. These studies they do not consider the effects of the third length scale in the system which is correlation length ξ. Although, some reports showed how correlation length plays effective role in wave scattering in Rayleigh-Rice framework[6,19-22], it could be useful to consider this effect in Kirchhoff theory, too. Schiffer indicated when ξ comparable with λ, the effect are losing in reflection, shifting in Brewster angle to smaller ones and slight reddening of scattering light[6] in rayleigh-Rice theory. Also, Yanguas-Gil et al.[22] showed when the correlation length is smaller than the wavelength (ξ < λ), the single parameter $\frac{w^2}{\xi^a}$ contains the information of wave scattering phenomena in Rayleigh-Rice framework.

These three length scales (λ, σ and ξ) depend on dimensions will affect the scattered wave. When $\lambda \gg \xi$, the correlation length has no effect on the scattered wave

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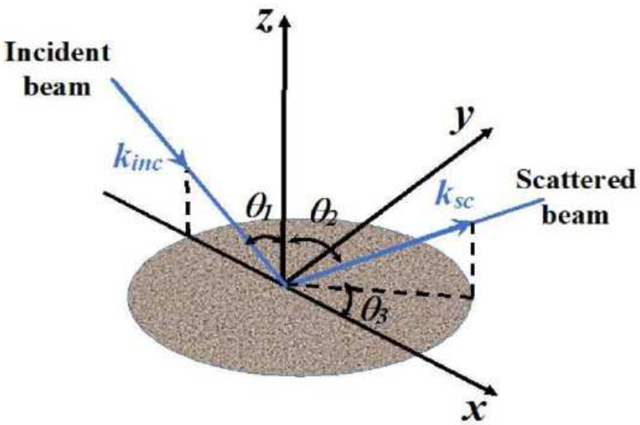


Figure 1. The geometry used for wave scattering from a rough surface. Here θ_1 is the angle between the incident beam and the normal of the surface. θ_2 is the scattered angle and θ_3 is the angle between the incident and scattered planes. k_{inc} and k_{sc} are the wave numbers for incident and scattered beam respectively.

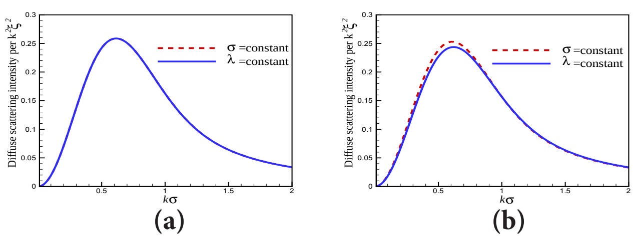


Figure 2. Dependence of diffuse scattered intensity per $k^2\xi^2$ on $k\sigma$ for angles $\theta_1 = \theta_3 = 0°$, $\theta_2 = 5°$ and $H = 1$, **(a)** $\lambda = 1000\xi$, **(b)** $\lambda = \xi$.

and $k\sigma$ is a suitable parameter to present wave scattering from the rough surface (see Figure 2(a)). But when ξ and λ are in the same range another dimensionless parameter $k\xi$ will appear as well. We need to know how $k\sigma$ is changed (see Figure 2(b)). To make any changes in $k\sigma$, we can either change k or σ. Different values of k or σ may result in the same value for $k\sigma$. In this paper, we focus on scattering problem in framework of Kirchhoff approximation when there are three length scales in the system, while ξ and λ are in the same range.

To clarify the concept of the relation between roughness, correlation length and wavelength in a scattering system we can represent the problem with a runner running on a surface made up of rocks with different sizes. The longitudinal size of a rock and the height of that indicate correlation length, ξ, and the height root mean squared, σ, respectively. The speed of the runner depends on the longitudinal size and the height root mean squared of the rocks in addition to the size of his shoes. Here the size of the runner shoes can be interpreted as observation scale or wavelength of incident beam. If the runner wants to run fast, the size of the rocks and the height mean squared of the rocks are important. Really small rocks with small height root mean squared have no effect in the speed of the runner. This situation in a scattering system is equivalent to the wavelength larger than the surface roughness (σ) which means the diffuse scattering intensity is small. On the other hand if the size of his shoes is comparable to the size of the rocks, he has to slow down to be able to go through the troughs made by rocks. If the height mean squared are big, the runner shoes will be trapped between the rocks that makes him to slow down. This means that by decreasing wavelength, the fluctuation of the surface is more intuitive and the diffuse scattering intensity increases. By decreasing the wavelength, the diffuse scattering intensity has some reduction. This resembles the situation where the sizes of the rocks or the height mean squared are larger than the size of the runner's shoes. In this situation the parameters of the rocks has less effect in the speed of the runner. This means if the wavelength is smaller than the fluctuations of the rough surface, the scattering intensity decrease. In addition to these parameters, the complexity of the surface roughness is another key determining point in the diffuse scattering intensity. This means a surface that is covered by the periodical peaks and troughs will have different diffuse scattering intensity from a surface with height fluctuations. This parameter of a rough surface is described as Hurst exponent, H.

2. Kirchhoff Theory

Any incident field can be written as $\psi^{inc}(r) = exp(-i\mathrm{k}_{inc}.\mathrm{r})$, where k_{inc} is the wave number of the incident beam and r is the position vector. The incident beam is scattered from a rectangular rough surface area with following conditions: $-X < x_0 \leq X$ and $-Y < y_0 \leq Y$. The scattered field from the mentioned rough surface is shown by ψ^{sc}. By using Kirchhoff theory one can determine the distribution of coherent and diffuse parts of scattered wave from a rough surfaces[3].

Kirchhoff theory is calculated based on three hypotheses[3]. (1) The roughness of each point of the surface is assumed to have the same optical behavior as its tangent plane. Fresnel laws can thus be locally applied. (2) The surface reflectivity R_0 is independent of the position on the rough surface and the local angle of incidence. (3)

Calculations are performed in the far-field, which works when the surface area S_0 is small. For all the calculations in this study, we assumed a monochromatic wave illuminates the rough surface with finite dimensions and R_0 reflectivity. According to Dirichlet boundary conditions, $R_0 = -1$ for such surfaces[3].

Through these assumptions we can obtain that the coherent part of scattered light wave is:

$$\psi^{sc}(r) = \frac{-ike^{ikr}}{4\pi r} 2F(\theta_1, \theta_2, \theta_3) \int_{S_M} exp[ik\phi(x_0, y_0)] dx_0 dy_0, \quad (1)$$

Where the phase function is $\phi(x_0 + y_0) = Ax_0 + By_0 + Ch(x_0, y_0), F = \frac{1}{2}\left(\frac{Aa}{C} + \frac{Bb}{C} + c\right)$ and

$$A = \sin\theta_1 - \sin\theta_2 \cos\theta_3$$

$$B = -\sin\theta_2 \sin\theta_3$$

$$C = -(\cos\theta_1 + \cos\theta_2)$$

$$a = \sin\theta_1 (1 - R_0) + \sin\theta_2 \cos\theta_3 (1 + R_0)$$

$$b = \sin\theta_2 \sin\theta_3 (1 + R_0)$$

$$c = \cos\theta_2 (1 + R_0) - \cos\theta_1 (1 - R_0)$$

The coherent field could be derived from the average amplitude of the scattered field and the intensity of the coherent field for a surface with a Gaussian height distribution,

$$I_{coh} = \langle\psi^{sc}\rangle\langle\psi^{sc*}\rangle = e^{-g} I_0, \quad (2)$$

where $g = k^2C^2\alpha^2$ and Io is the coherent scattered intensity from a smooth surface with the same size as the rough surface.

The average diffuse field intensity can also be calculated using ψ^{sc},

$$\langle I_d\rangle = \langle\psi^{sc}\psi^{sc*}\rangle - \langle\psi^{sc}\rangle\langle\psi^{sc*}\rangle$$

$$= \frac{k^2F^2}{2\pi r^2} A_M e^{-g} \int_0^\infty J_0\left(kR\sqrt{A^2 + B^2}\right)\left[e^{gCor(R)} - 1\right] RdR, \quad (3)$$

where $A_M = 4XY$ is the area of the mean scattering surface and $Cor(R)$ is the height-height correlation function and it is defined as $Cor(R) = \frac{\langle h(x+R)h(x)\rangle}{\sigma^2}$.

3. Scattering from Self-affine Surface

One of the main groups of the rough surface is represented by self-affine fractal scaling, defined by Mandelbrot in terms of fractional Brownian motion[23]. Lets consider a rough surface with a single-valued height function, $z(r)$ of the in-plane positional vector, $r = (x, y)$. All rough surfaces are characterized by two parameters, the mean-square roughness $\sigma = < z(r)^2 >^{1/2}$ and $z(r) = h(r) - < h(r) >$. As mentioned earlier, $h(r)$ is the height function and $< \cdots >$ is the spacial average over a planar reference surface. We assume that $z(r) - z(r')$ is a stochastic Gaussian variable whose distribution depends on the difference of the two position vectors r and r′, $(x' - x, y' - y)$. Using the height function we can define the structure function as $S(R) = <[z(r') - z(r)]^2>$, where $R = | r' - r |$. The average is taken over all pairs of points on the surface that are separated horizontally by the distance R. $S(R)$ could be defined in terms of the height-height correlation function that was defined earlier, $Cor(R) = \frac{< z(R)z(0) >}{\sigma^2}$, as,

$$g(R) = 2 < z(r)^2 > -2 < z(r)z(r') > = 2\sigma^2 - 2 < z(R)z(0) > = 2\sigma^2 - 2\sigma^2 Cor(R). \quad (4)$$

If the surface exhibits self-affine roughness, $S(R)$ will scale as $S(R) \propto \alpha\, R^{2H}$, [24], where $0 < H < 1$ is referred to as Hurst exponent [25]. The Hurst exponent represents the degree of the surface irregularity.

Large values of H correspond to smooth height-height fluctuations, while the small values shows the jagged and irregular surfaces at the short length scale. The mean-Square Roughness, $S(R)$, of any physical self-affine surface will saturate at sufficiently large horizontal lengths. Thus $S(R)$ is characterized by a correlation length,

$$S(R)\, \alpha R^{2H}, For\ R \ll \xi,$$

$$S(R) = 2\sigma^2, For\ R \gg \xi \quad (5)$$

By using the diffuse reflectivity data[26,27] the correlation function for a self-affine surfaces can be written as[28],

$$Cor(R) = e^{-(R/\xi)^{2H}} \tag{6}$$

When $R \gg \xi$, there will be no correlation, $Cor(R)$ = 0 and for $R \ll \xi$ correlation function can be written as $Cor(R) \simeq 1 - \left(\frac{R}{\xi}\right)^{2H}$ This indicates for short length scale $r \ll \xi$, the correlation function shows power-law behavior[29].

At the largest possible value for Hurst exponent, smooth height-hight fluctuations, the correlation function is Gaussian and Eq. 3 for the diffuse scattered intensity can be written as[3]:

$$< I_d >= \frac{k^2 F^2 \xi^2 e^{-g}}{4\pi r^2} A_M \sum_{n=1}^{\infty} \frac{g^n}{n!n} exp\left(\frac{k^2\left(A^2 + B^2\right)\xi^2}{4n}\right) \tag{7}$$

For a Gaussian noise surface with H = 0.5, we can obtain the following expression for the diffuse scattered intensity from Eq. 3[18]:

$$< I_d >= \frac{k^2 F^2}{2\pi r^2} A_M e^{-g} \sum_{n=1}^{\infty} \frac{g^n}{n!} \frac{\xi^2}{n^2\left(1 + \frac{k^2\left(A^2 + B^2\right)\xi^2}{n^2}\right)^{\frac{3}{2}}}. \tag{8}$$

4. Result and Discussion

It was stated earlier, when the system is characterized by the three length scales k, σ, and ξ, the dimensionless parameter $k\sigma$ is unable to give a complete description of the scattering problem from the rough surface. Hence, a new dimensionless parameter $k\xi$ is introduced which enables a comprehensive study on the scattering phenomena. Hence, the combined effects of the wavelength, roughness and correlation length on the scattered waves need to be studied. The formalism presented here would provide basis for both theoretical studies and experimental observations, in a sense that two approaches are proposed based on the physical applications. To comply with the theoretical modeling needs, the chosen variable would be $k\xi$ which is more convenient due to its dimensionless nature, and to comply with the observers needs in order to calculate the scattered intensity, values of the wave-length and roughness have been chosen close to expected measurements. These two approaches are presented in the following sections.

4.1 Theoretical Approach

The scattered intensity depends on the altitude correlation function of the rough surface, so the area of the correlated section (the part of the surface that has correlated altitudes) also plays an important role. To calculate the intensity of the scattered wave, we use r = 0 and r as the lower and upper limits of the integral respectively, where r and ξ have the same order of magnitude. For the case $r \ll \xi$, the correlated function and the integrand will be zero. On the other hand we know that a correlated surface is proportional to ξ^2.

Also we should keep in mind that ξ is a characteristic length of the surface which possesses a scaling behavior. This implies consideration of the scale of observation; this indicates the importance of the incident wave length to the surface. Consequently the parameter $k\xi$ would show its importance. Since the scattered field intensity depends on ξ^2, in order to gain its independence from the parameters ξ and λ, it is divided by $k^2\xi^2$. In such a condition, we obtained an expression independent of the parameters ξ, σ, λ. When the correlated regime is much smaller than the wave length, the intensity only depends on the $k\sigma$ (see Figure 2(a)). However when ξ is as of the order of λ its effects on the scattered wave intensity becomes prominent and the curve indicating the variations of the diffused intensity in terms of $k\sigma$ becomes obviously dependent on k or σ (see Figure 2(b)).

4.2 Experimental Approach

In this section we study the diffuse scattering intensity with the presence of three length scale for rough surfaces. We suppose that ξ and λ are in comparable range. As mentioned above to change $k\sigma$ we can either change k or σ. We discuss about the effects of changing each of these parameters individually. Furthermore, we will show our results with different values of correlation length, ξ, and Hurst exponent, H, on self-affine surfaces.

4.2.1 Effect of Variation in kσ

Figure 3 and Figure 4 show the diffuse scattering intensity as a function of scattering angle for different $k\sigma$ at two different values of Hurst exponent. In the former one the wavelength is constant, $\lambda = 633nm$ and the σ is variable and in the latter one the height root mean squared is constant,

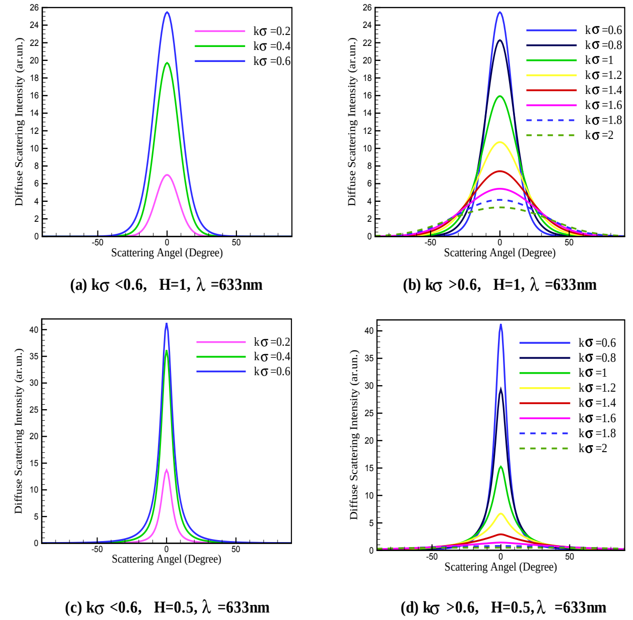


Figure 3. The diffuse scattering intensity as a function of scattering angle with constant λ = 633*nm*. The graphs are plotted for values of $k\sigma$ and Hurst exponent. The incident angle and the angle between the incident and scattered planes are equal, $\theta_1 = \theta_3 = 0°$ and the correlation length is $\xi = 1000nm$.

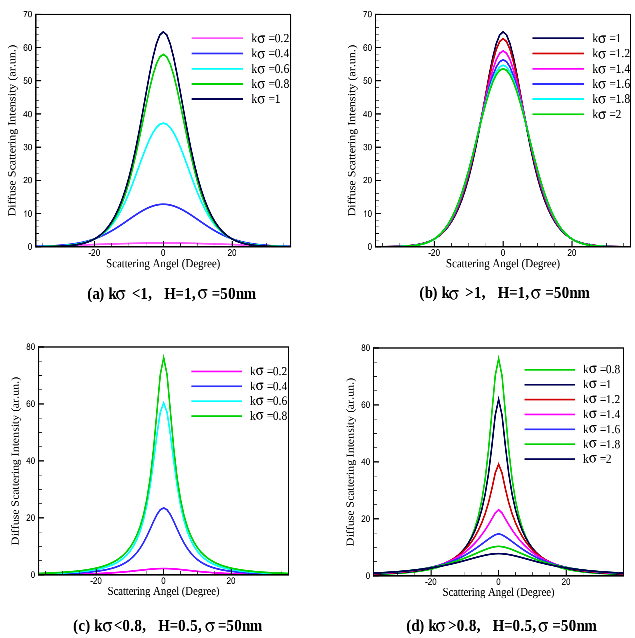


Figure 4. The diffuse scattering intensity as a function of scattering angle with constant $\sigma = 50nm$. The graphs are plotted for different values of $k\sigma$ and Hurst exponent. The incident angle and the angle between the incident and scattered planes are equal and $\theta_1 = \theta_3 = 0°$ and the correlation length is $\xi = 1000nm$

$\sigma = 50$ nm, when the wavelength can modify as a variable. In both Figures we found the diffuse scattering intensity at different values of $k\sigma$ between 0.2 and 2.0 with the intervals of 0.2. These values are in agreement with experimental values in previous works[13,14].

In both figures the diffuse scattering intensity is calculated for two different values of Hurst exponent. By comparing the two graphs on the top ($H = 1$) with the ones on the bottom ($H = 0.5$), we can see that the diffuses scattering intensity has smaller standard deviation at $H = 0.5$. Also, the peak of the diffuse scattering intensity is higher for smaller value of the Hurst exponent.

In all different cases, we found a threshold value for $k\sigma$ where the diffuse scattering intensity has its own maximum value. This threshold value is happening where the correlation length, height mean squared and the wavelength are in the same order of magnitude. In both Figure 3 and Figure 4, the graphs on the left and right show the diffuse scattering intensity for the $k\sigma$ values smaller and larger than the threshold value respectively. The rate of changing the diffuse scattering intensity is larger for the $k\sigma$ values smaller than the threshold value.

In Figure 3, where the wavelength is constant, the standard deviation of diffuse scattering intensity is getting larger by increasing $k\sigma$. However when the height mean squared is constant (σ in Figure 4) the standard deviation of diffuse scattering intensity decreases as $k\sigma$ increases. Furthermore, by decreasing Hurst exponent due to the increased irregularity of the surface roughness, the diffuse intensity increases. The curve of the diffuse intensity is getting sharper and the standard deviation of that decreases.

In some of the studies the scattering intensity has been studied as a function of dimensionless $k\sigma$. As discussed before the changes in $k\sigma$ could be the result of variation in the wavelength or the height mean squared parameters. Our result show that not only the value of $k\sigma$ is one of the key parameters, but also how this parameter has been changed is important too. Our results show that the scattered intensity varies by changing the wavelength and the height mean squared individually. In another word, studying the diffuse scattering intensity as a function of $k\sigma$ does not depict all the features of the scattered intensity and we need to consider the effects of the wavelength and the height mean squared as well.

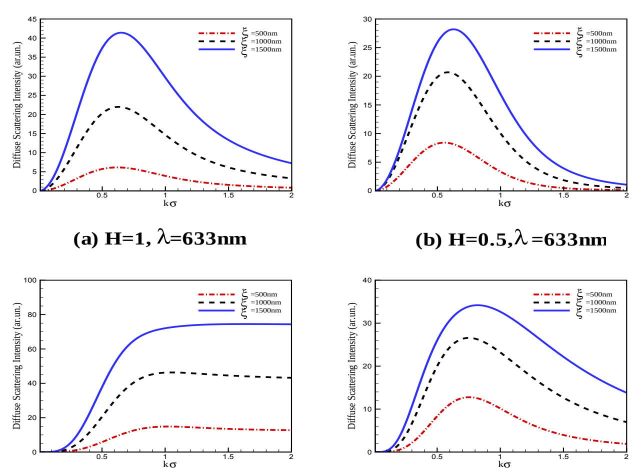
Furthermore, by adding more parameters to our system such as correlation length, ξ, other dimensionless parameter such as $k\xi$ effects the scattering intensity. We will discuss the effect of this parameter in the next section. To increase

$k\sigma$ while the wavelength is constant, both the height mean squared, σ, and the correlation length, ξ, should change to keep the Hurst exponent constant. If the variation in $k\sigma$ is due to the changes in the wavelength, both the height mean squared, σ, and the correlation length, ξ, remain constant for a constant value of the Hurst exponent.

4.2.2 Effect of correlation Length

Figure 5 shows the diffuse scattering intensity as a function of $k\sigma$ for two different values of the Hurst exponent H = 0.5, 1.0. For the top graphs λ is constant and for the ones at the bottom σ is constant. In all four graphs of Figure 5, the diffuse scattering intensity is calculated for three values of the correlation length, ξ = 500, 1000, 1500 nm. Besides, for all the graphs the angle between the incident beam and the normal of the plane and the angle between the incident and scattered planes are kept constant, $\theta_1 = \theta_3 = 0°$. The diffuse scattered intensity is calculated for one specific scattered angle, $\theta_2 = 5°$.

In Figure 5 (a and b) the diffuse scattering intensity increases as $k\sigma$ increases due to the increment of σ. Both of these graphs show a maximum value for intensity at a specific value of $k\sigma$ that is the same as the threshold value mentioned in the previous section. Similar behavior can be seen in Figure. 5 (d) where σ is kept constant and $k\sigma$ increases be decreasing λ. However, in Figure 5 (c), the intensity ends up to a plateau after reaching to its maximum value. This result is confirmed by Figure 4 (b), where shows that the changes of the diffuse scattering intensity is small at $\theta_2 = 5°$.



(a) H=1, λ=633nm **(b) H=0.5,λ=633nm**

(c) H=1,σ=50nm **(d) H=0.5,σ=50nm**

Figure 5. Dependence of diffuse scattered intensity on $k\sigma$ for different correlation length (ξ = 500, 1000, 1500*nm*) and for angles $\theta_1 = \theta_3 = 0°$, $\theta_2 = 5°$.

Even though the parameters that we used to calculate the diffuse scattering intensity is different in the top and the bottom graphs, they show the similar behavior when $k\sigma$ increases. This is due to the relative magnitude of the wavelength and the roughness of the surface. In the top graphs λ is constant for the top graphs, but σ has to increase to enlarge $k\sigma$. This means the wavelength is decreasing compare to σ. On the other hand, σ is constant in the bottom graphs, so to enlarge $k\sigma$, λ has to decrease. This also results in small wavelength compare to the roughness of the surface. When the wavelength is smaller than the roughness of a surface, the diffuse scattering intensity decreases. This is similar to the runner example that we mentioned earlier. If we assume that the Hurst exponent of a surface is constant, the changes in correlation length, ξ, is coupled with that of the height mean squared, σ. Our results show that the scattering intensity for a surface with constant Hurst exponent not only depends on the wavelength of the incident beam, but also it changes by changing the σ and ξ.

5. Conclusion

The scattered wave from a rough surfaces depends on the roughness parameters such as σ, ξ and λ. In the limiting case of $\lambda \gg \xi$, only two characteristic lengths; λ which is the observation scale, and σ which is the scale of surface, or their ratio comes in to effect on the scattered wave intensity. This means that in this case $k\sigma$ is important. This limit has been the case of consideration by other researchers.

But if ξ and λ are in comparable range, the system possesses three characteristic lengths; λ the observation scale, σ and ξ which are the scales of the rough surface. In this case, $k\sigma$ does not prove to be a suitable parameter in order to present wave scattering from the rough surface, unless the consideration of a varying k or σ is studied for a constant $k\sigma$. In other words is must be understood that which of the parameters k or σ is varying and which is constant. This has been emphasized in this work. Besides, we study the scattering phenomena including all three length scales, k, σ and ξ, both theoretically and experimentally.

To study the relation of the scattered wave intensity and $k\sigma$ when ξ and λ are comparable, we used a self-affine rough surface. We compare two regimes for the $k\sigma$ parameter, one is σ = cont. and wavelength is changed and the other is λ = *const.* and σ is changed. The result shows the diffuse scattering intensity in both of them have a threshold value for $k\sigma$ which happening when the correlation length, height mean squared and wave length have comparable value. It is noted that by existence of correlation length ξ, $k\sigma$ is not the only dimensionless parameter. Therefore we must consider by variation of ξ which parameters (k or σ) are being changed.

The role of the Hurst exponent has been investigated for self-affine rough surfaces. Such an examination was performed over a wide range of surface topographies, from logarithmic ($H = 0$) to a power-law self-affine rough surface, $0 < H < 1$. The roughness exponent H has a strong impact on the diffused part of wave scattering mainly for relatively large correlation lengths. Therefore, Hurst exponent must be taken carefully into account before deducing the roughness correlation lengths from wave scattering measurements.

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