

Stability Analysis of a Vector-borne Disease Model with Nonlinear and Bilinear Incidences

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Abstract

Background/Objectives: To formulate and analyze a vector host epidemic model with non-monotonic and bilinear incidences. **Methods/Statistical Analysis:** The stability conditions of disease free equilibrium and endemic equilibrium are investigated by constructing suitable Lyapunov functions. Numerical simulation is carried out to justify the theoretical results. **Results/Findings:** The disease becomes endemic when the basic reproduction number is greater than one and it fades out when it is less than one. **Conclusion/Application:** In endemic state of the disease, number of infective host decreases as awareness of vaccination and preventive measures increases and number of vectors approaches zero as the awareness of use of insecticides and cleanliness tends to infinity.

Keywords: Basic Reproduction Number, Disease Free Equilibrium, Endemic Equilibrium, Stability, Vector Borne Diseases

1. Introduction

Vectors become the main mediators of infectious diseases to the host population¹. Awareness about preventive measures to reduce vector-host contacts² and the insecticide control of the vector helps to eradicate the disease. Infectious diseases have major social and economical impact in the population. Mathematical models on epidemiology provide significant insight into population behavior and control. They play an important role in understanding the dynamics of the diseases and to make the public health policies for controlling disease³.

Our aim is to develop a vector born disease model with non-monotonic incidence rate and bilinear incidence rate⁴ which describes the effect of social awareness on epidemics. In section 2, we formulate a system of differential equations which represent the vector host epidemiological model with non-monotonic rate and bilinear incidence rate and compute the basic reproduction number. We also obtain the disease free equilibrium and endemic equilibrium for the model and analyze the stability conditions for these equilibriums in section 3. In section 4, we show the outcomes using numerical simulation.

2. Model Formulation

Let $S(t)$ be the number of susceptible individuals, $I(t)$ be the number of infective individuals, and $R(t)$ be the number of recovered individuals in host population at time t such that $N_1(t) = S(t) + I(t) + R(t)$. We take α_1 is the recruitment rate of host population, λ_1 is the transmission rate from vector to host, β_1 is natural death rate of host population, γ is per capita recovery rate of host, a is the parameter which measures the effect of awareness of vaccination and preventive measures among the host population and b is the parameter which measures the effect of cleanliness and use of insecticide for vector control. We have chosen the parameter a so that $1 + aV + bV^2 > 0$ for all $V \geq 0$ and the parameter b be positive. Let $M(t)$ be the number of susceptible vectors and $V(t)$ is the number of infective vectors at time t such that $N_2(t) = M(t) + V(t)$. We take α_2 is the recruitment rate of vector population, β_2 is the natural death rate of vector population and λ_2 is the transmission rate from host to vector.

Hence, our vector host epidemiological model with non-monotonic incidence rate for hosts and the bilinear incidence rate for vectors is represented by a system of differential equations as follows:

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$$\begin{aligned}
 \frac{dS(t)}{dt} &= \alpha_1 - \frac{k\lambda_1 S(t)V(t)}{1+aV(t)+bV^2(t)} - \beta_1 S(t) \\
 \frac{dI(t)}{dt} &= \frac{k\lambda_1 S(t)V(t)}{1+aV(t)+bV^2(t)} - \gamma I(t) - \beta_1 I(t) \\
 \frac{dR(t)}{dt} &= \gamma I(t) - \beta_1 R(t) \\
 \frac{dM(t)}{dt} &= \alpha_2 - \lambda_2 M(t)I(t) - \beta_2 M(t) \\
 \frac{dV(t)}{dt} &= \lambda_2 M(t)I(t) - \beta_2 V(t)
 \end{aligned} \quad (2.1)$$

Therefore we get

$$\begin{aligned}
 \frac{d}{dt} N_1(t) &= \alpha_1 + \beta_1 N_1(t) \\
 \frac{d}{dt} N_2(t) &= \alpha_2 + \beta_2 N_2(t)
 \end{aligned}$$

The total population sizes of host and vector are asymptotically constant.

$$\text{i.e. } \lim_{t \rightarrow \infty} N_1(t) = \frac{\alpha_1}{\beta_1}, \quad \lim_{t \rightarrow \infty} N_2(t) = \frac{\alpha_2}{\beta_2}$$

Without loss of generality, we assume that $N_1(t) = \frac{\alpha_1}{\beta_1}$, $N_2(t) = \frac{\alpha_2}{\beta_2}$ for all $t \geq 0$.

Therefore the dynamical system (1.1) is now equivalent to the dynamical system given by

$$\begin{aligned}
 \frac{dS(t)}{dt} &= \alpha_1 - \frac{k\lambda_1 S(t)V(t)}{1+aV(t)+bV^2(t)} - \beta_1 S(t) \\
 \frac{dI(t)}{dt} &= \frac{k\lambda_1 S(t)V(t)}{1+aV(t)+bV^2(t)} - \gamma I(t) - \beta_1 I(t) \\
 \frac{dV(t)}{dt} &= \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V(t) \right) I(t) - \beta_2 V(t)
 \end{aligned} \quad (2.2)$$

The values of R and M are determined from $R = \frac{\alpha_1}{\beta_1} - S - I$ and $M = \frac{\alpha_2}{\beta_2} - V$, respectively.

We need non-negative solutions for biological reasons. Due to mathematical properties of the solutions we study the system (2.2) in the closed set

$$\Gamma = \left\{ (S, I, V) \in \mathbb{R}_+^3 : S + I \leq \frac{\alpha_1}{\beta_1}, 0 \leq V \leq \frac{\alpha_2}{\beta_2}, S \geq 0, I \geq 0 \right\}$$

System (2.2) has disease free equilibrium $E_0 \left(\frac{\alpha_1}{\beta_1}, 0, 0 \right)$.

To find the endemic equilibrium of system (2.2), set

$$\begin{aligned}
 \alpha_1 - \frac{k\lambda_1 S(t)V(t)}{1+aV(t)+bV^2(t)} + \beta_1 S(t) &= 0 \\
 \frac{k\lambda_1 S(t)V(t)}{1+aV(t)+bV^2(t)} - \gamma I(t) - \beta_1 I(t) &= 0 \\
 \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V(t) \right) I(t) - \beta_2 V(t) &= 0
 \end{aligned}$$

This gives

$$\begin{aligned}
 b\beta_1\beta_2^2(\gamma + \beta_1)V^2 + [k\alpha_1\beta_2\lambda_1\lambda_2 + k\lambda_1\beta_2^2(\gamma + \beta_1) + a\beta_1\beta_2^2(\gamma + \beta_1)]V \\
 + \beta_1\beta_2^2(\gamma + \beta_1) - k\alpha_1\alpha_2\lambda_1\lambda_2 = 0
 \end{aligned}$$

The basic reproduction number is defined as follows

$$R_0 = \frac{k\alpha_1\alpha_2\lambda_1\lambda_2}{\beta_1\beta_2^2(\gamma + \beta_1)} \quad (2.3)$$

The endemic equilibrium $E^*(S^*, I^*, V^*)$ is given by the following equations

$$S^* = \frac{(\gamma + \mu_1)\beta_2^2(1+aV^*+bV^{*2})}{k\lambda_1\lambda_2[\alpha_2 - \beta_2V^*]} \quad (2.4)$$

$$I^* = \frac{\beta_2^2V^*}{\lambda_2[\alpha_2 - \beta_2V^*]} \quad (2.5)$$

$$V^* = \frac{-\Delta + \sqrt{\Delta^2 - 4b\beta_1\beta_2^2(\gamma + \beta_1)^2(1-R_0)}}{2b\beta_1\beta_2^2(\gamma + \beta_1)} \quad (2.6)$$

where $\Delta = k\alpha_1\beta_2\lambda_1\lambda_2 + k\lambda_1\beta_2^2(\gamma + \mu_1) + \alpha_1\beta_1\beta_2^2(\gamma + \beta_1)$

3. Stability Analysis

In this section, we derive the stability conditions for the disease-free and the endemic equilibrium of model (2.2).

3.1 Theorem

If $R_0 < 1$ then the disease free equilibrium E_0 is locally asymptotically stable and it is unstable for $R_0 > 1$.

Proof: For the disease free equilibrium E_0 , we obtain the Jacobian matrix as:

$$J(E_0) = \begin{bmatrix} -\beta_1 & 0 & -k\lambda_1 \frac{\alpha_1}{\beta_1} \\ 0 & -\gamma - \beta_1 & k\lambda_1 \frac{\alpha_1}{\beta_1} \\ 0 & \lambda_2 \frac{\alpha_2}{\beta_2} & -\beta_2 \end{bmatrix} \quad (3.1)$$

Characteristic equation of this matrix is given by

$$(\xi + \beta_1) \left[\xi^2 + (\gamma + \beta_1 + \beta_2) \xi + (\gamma + \beta_1) \beta_2 (1 - R_0) \right] = 0$$

Clearly, this characteristic equation has a root $\xi = -\beta_1$ which is negative. Another two roots are determined by the equation

$$\xi^2 + (\gamma + \beta_1 + \beta_2) \xi + (\gamma + \beta_1) \beta_2 (1 - R_0) = 0$$

By Routh - Hurwitz criterion^{5,6}, we have that if $R_0 < 1$ then above equation has both the roots with negative real part and if $R_0 > 1$ then it has one of the root with negative real part and another with positive real part. Thus, we obtain that all the Eigen values of the above characteristic equation have negative real part for $R_0 < 1$ and hence local asymptotically stability of the disease free equilibrium E_0 is established. When $R_0 > 1$, two of the Eigen values are negative and one is positive and so E_0 is unstable in this case. Hence the theorem follows.

3.2 Theorem

The disease free equilibrium E_0 is globally asymptotically stable for $R_0 \leq 1$ and if $R_0 > 1$ then it is unstable.

Proof: We consider the function L_1 as follows,

$$L_1 = \frac{k\lambda_1 \alpha_1}{\beta_1 \beta_2} V + I$$

$$\frac{dL_1}{dt} = \frac{k\lambda_1 \alpha_1}{\beta_1 \beta_2} \left[\lambda_2 \left(\frac{\alpha_2}{\beta_2} - V \right) I - \mu_2 V \right] + \frac{k\lambda_1 S V}{1 + aV + bV^2} - (\gamma + \beta_1) I$$

$$= -(\gamma + \beta_1) \left[1 - \frac{k\alpha_1 \alpha_2 \lambda_1 \lambda_2}{\beta_1 \beta_2^2 (\gamma + \beta_1)} \right] I - \frac{k\alpha_1 \lambda_1 \lambda_2}{\beta_1 \beta_2} V I - \frac{k\alpha_1 \lambda_1}{\beta_1} V + \frac{k\lambda_1 S V}{1 + aV + bV^2}$$

$$\leq -(\gamma + \beta_1) [1 - R_0] I - \frac{k\alpha_1 \lambda_1 \lambda_2}{\beta_1 \beta_2} V I, \quad \text{as } S \leq \frac{\alpha_1}{\beta_1}$$

Since at all $t \geq 0$, if $R_0 \leq 1$ then $\frac{dL_1}{dt} \leq 0$, the function L_1 is a Lyapunov function. Also, at the disease free equilibrium i.e. at $E_0 \left(\frac{\alpha_1}{\beta_1}, 0, 0 \right)$ we find $\frac{dL_1}{dt} = 0$. Thus, $\{E_0\}$ is the

largest invariant set in the closed set Γ . Therefore, E_0 is globally stable using LaSalle's invariance principle⁷.

We now investigate the local stability of endemic equilibrium and derive the global stability of endemic equilibrium in the feasible region Γ by proving uniform persistence of the system (2.2). We also apply theory of second compound equations to show asymptotic orbital stability of periodic solutions.

3.3 Theorem

The endemic equilibrium E^* of the system (2.2) is locally asymptotically stable if $R_0 > 1$.

Proof: We have

$$-\frac{k\lambda_1 V^*}{1 + aV^* + bV^{*2}} - \beta_1 & 0 \text{ \& } -\frac{k\lambda_1 S^* (1 - bV^{*2})}{(1 + aV^* + bV^{*2})^2} @ \frac{k\lambda_1 V^*}{1 + aV^* + bV^{*2}} \\ & \text{\& } -(\gamma + \beta_1) \text{ \& } \frac{k\lambda_1 S^* (1 - bV^{*2})}{(1 + aV^* + bV^{*2})^2} @ 0 \text{ \& } \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V^* \right) \text{ \& } -\lambda_0$$

$$trJ(E^*) = -\frac{k\lambda_1 V^*}{1 + aV^* + bV^{*2}} - 2\beta_1 - \gamma - \lambda_2 I^* - \beta_2 < 0 \quad (3.2)$$

Using the value of S^* , we have

$$\mu_2 (\gamma + \beta_1) = \frac{k\lambda_1 \lambda_2 S^*}{(1 + aV^* + bV^{*2})^2} \left(\frac{\alpha_2}{\beta_2} - V^* \right)$$

Also

$$\det J(E^*) = - \left(\frac{k\lambda_1 V^*}{1 + aV^* + bV^{*2}} + \beta_1 \right) (\gamma + \beta_1) (\lambda_2 I^* + \beta_2) \\ - \frac{k\lambda_1 \beta_1 S^* (bV^{*2} - 1)}{(1 + aV^* + bV^{*2})^2} \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V^* \right)$$

$$\therefore \det J(E^*) < 0, \quad \text{provided } V^{*2} > \frac{1}{b} \quad (3.3)$$

Second additive compound matrix $J^2(E^*)$ is given by

$$J^\uparrow [2](E^\uparrow \star) = \blacksquare (k\lambda_1 V^\uparrow \star) / (1 + a\blacksquare V^\uparrow \star + bV^\uparrow \star \blacksquare^\uparrow 2) - \blacksquare 2\beta_1 \blacksquare_\downarrow 1 \\ - \gamma \text{ \& } (\blacksquare k\lambda_1 \blacksquare_\downarrow 1 S^\uparrow \star (1 - b\blacksquare V^\uparrow \star \blacksquare^\uparrow 2)) / (1 + a\blacksquare V^\uparrow \star + bV^\uparrow \star \blacksquare \gamma + \beta_1 + \lambda_{\blacksquare_\downarrow})$$

We have, $\text{tr } J(E^*)$, $\det J(E^*)$ and $\det J^2(E^*)$ all negative. Hence, all Eigen values of $J(E^*)$ have negative real part^{8,9}. Hence the theorem.

3.4 Theorem

If $R_0 > 1$, then system (2.2) is uniformly persistent, that is, there exists $\epsilon > 0$ (independent of initial conditions), such that $\liminf_{t \rightarrow \infty} S(t) > \epsilon$, $\liminf_{t \rightarrow \infty} I(t) > \epsilon$ and $\liminf_{t \rightarrow \infty} V(t) > \epsilon$.

Proof: To prove this, we show the following results:

- For system (2.2), E_0 is only one omega-limit point on the boundary of Γ .
- For $R_0 > 1$, E_0 cannot be the omega-limit point of any orbit in $\text{Int } \Gamma$.

(i) The vector field is transversal to the boundary of Γ , except in the S-axis, which is invariant for the system (2.2). On the S-axis, we have

$$\frac{dS}{dt} = \alpha_1 - \beta_1 S$$

It follows from the above expression that $S \rightarrow \alpha_1/\beta_1$ as $t \rightarrow \infty$. So, the first part is proved.

(ii) Now we define the following function in Γ .

$$\begin{aligned} L_2(t) &= \frac{\beta_1 \beta_2 (1+R_0)}{2k\lambda_1 \alpha_1} I(t) + V(t) \\ \frac{dL_2(t)}{dt} &= \frac{\beta_1 \beta_2 (1+R_0)}{2k\lambda_1 \alpha_1} \left(\frac{k\lambda_1 S(t)V(t)}{1+aV(t)+bV^2(t)} - \gamma I(t) - \beta_1 I(t) \right) \\ &\quad + \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V(t) \right) I(t) - \beta_2 V(t) = \frac{\beta_1 \beta_2 (1+R_0)}{2\alpha_1} \frac{S(t)V(t)}{(1+aV(t)+bV^2(t))} \\ &\quad - \beta_2 V(t) + \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V(t) \right) I(t) - \frac{(1+R_0)}{2R_0} \frac{\lambda_2 \alpha_2}{\beta_2} I(t) \\ &\geq \frac{\beta_1 \beta_2 (1+R_0)}{2\alpha_1} \left[S(t) - \frac{2\alpha_1}{\beta_1 (1+R_0)} \right] V(t) + \lambda_2 \left[\frac{\alpha_2}{\beta_2} - V(t) - \frac{1}{2} \left(\frac{1}{R_0} + 1 \right) \frac{\alpha_2}{\beta_2} \right] I(t) \end{aligned}$$

Since $R_0 > 1$, then $\frac{1}{2} \left(\frac{1}{R_0} + 1 \right) < 1$ and $\frac{2}{(1+R_0)} < 1$. Hence, there

exists a neighborhood U of E_0 such that for $(S, I, V) \in \bigcup \cap \text{Int } \Gamma$, the expressions $S(t) - \frac{2\alpha_1}{\beta_1 (1+R_0)}$ and

$$\frac{\alpha_2}{\beta_2} - V(t) - \frac{1}{2} \left(\frac{1}{R_0} + 1 \right) \frac{\alpha_2}{\beta_2}$$

are positive. We have that $\frac{dL_2(t)}{dt} > 0$ in the set $U(E_0) - \{E_0\}$. Now the level sets of L

are the plane

$$\frac{\beta_1 \beta_2 (1+R_0)}{2k\lambda_1 \alpha_1} I(t) + V(t) = C$$

which move away from the S-axis when C increases. Since L_2 is increasing along the orbits starting in $\bigcup \cap \text{Int } \Gamma$,

all solutions of system (2.2) move away from E_0 .

3.5 Theorem

When $R_0 > 1$, the endemic equilibrium E^* is globally asymptotically stable.

Proof: The system (2.2) is uniformly persistent, and E^* is locally asymptotically stable for $R_0 > 1$. To prove this theorem, we prove that the system (2.2) has the property of stability of periodic orbits¹⁰.

Let $P(t) = (S(t), I(t), V(t))$ be a periodic solution of system (2.2). To prove the stability of periodic orbits, it is sufficient to prove that the following linear non-autonomous system,

$$W'(t) = (J^{[2]}P(t))W(t)$$

is asymptotically stable. The second additive compound matrix is given by

$$1 + aV + bV^2$$

For the solution $P(t)$, we have,

$$\begin{aligned} W_1'(t) &= - \left(\frac{k\lambda_1 V}{1+aV+bV^2} + 2\beta_1 + \gamma \right) W_1(t) + \frac{k\lambda_1 S(1-bV^2)}{(1+aV+bV^2)^2} W_2(t) \\ &\quad + \frac{k\lambda_1 S(1-bV^2)}{(1+aV+bV^2)^2} W_3(t) \\ W_2'(t) &= \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V \right) W_1(t) - \left(\frac{k\lambda_1 V}{1+aV+bV^2} + \beta_1 + \lambda_2 I + \beta_2 \right) W_2(t) \\ W_3'(t) &= \frac{k\lambda_1 V}{1+aV+bV^2} W_2(t) - (\gamma + \beta_1 + \lambda_2 I + \beta_2) W_3(t) \end{aligned} \quad (3.5)$$

To prove that system (3.5) is asymptotically stable, we consider the following function

$$L_3(W_1(t), W_2(t), W_3(t), S(t), I(t), V(t)) = \left\| W_1(t), \frac{I(t)}{V(t)} W_2(t), \frac{I(t)}{V(t)} W_3(t) \right\|$$

Where $\|\cdot\|$ is the norm in $\mathbb{R}^3$ defined by

$$\|W_1(t), W_2(t), W_3(t)\| = \sup \{ |W_1|, |W_2 + W_3| \}$$

From theorem (3.4), we obtain that the orbit of $P(t)$ remains at a positive distance from the boundary of Γ . There exists constant $c > 0$ such that

$$L_3(W_1(t), W_2(t), W_3(t), S(t), I(t), V(t)) \geq \|W_1(t), W_2(t), W_3(t)\| \quad (3.6)$$

Let $(W_1(t), W_2(t), W_3(t))$ be a solution of the system (3.5) and

$$L_3(t) = \sup \left\{ |W_1(t)|, \frac{I(t)}{V(t)} |W_2(t) + W_3(t)| \right\} \quad (3.7)$$

Thus, we obtain the following inequalities

$$\begin{aligned} D_+ |W_1(t)| &\leq -\left(\frac{k\lambda_1 V}{1+aV+bV^2} + 2\beta_1 + \gamma \right) |W_1(t)| + \frac{k\lambda_1 S(1-bV^2)}{(1+aV+bV^2)^2} \frac{V}{I} \left(\frac{I}{V} |W_2(t) + W_3(t)| \right) \\ D_+ |W_2(t)| &\leq \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V \right) |W_1(t)| - \left(\frac{k\lambda_1 V}{1+aV+bV^2} + \beta_1 + \lambda_2 I + \beta_2 \right) |W_2(t)| \\ D_+ |W_3(t)| &\leq \frac{k\lambda_1 V}{1+aV+bV^2} |W_1(t)| - (\gamma + \beta_1 + \lambda_2 I + \beta_2) |W_3(t)| \end{aligned} \quad (3.8)$$

Also, we have

$$D_+ |W_2(t) + W_3(t)| \leq \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V \right) |W_1(t)| - (\beta_1 + \lambda_2 I + \beta_2) |W_2(t) + W_3(t)|$$

Thus we obtain

$$\begin{aligned} D_+ \left(\frac{1}{V} |W_2(t) + W_3(t)| \right) &= \left(\frac{I'}{I} - \frac{V'}{V} \right) \frac{I}{V} |W_2(t) + W_3(t)| + \frac{I}{V} D_+ |W_2(t) + W_3(t)| \\ &\leq \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V \right) \frac{I}{V} |W_1(t)| + \left[\frac{I'}{I} - \frac{V'}{V} - \beta_1 - \lambda_2 I - \beta_2 \right] \frac{I}{V} |W_2(t) + W_3(t)| \end{aligned} \quad (3.9)$$

From the first equation of system (3.8) and equation (3.9), we get

$$D_+ L_3(t) \leq \sup \{ g_1(t), g_2(t) \} W(t)$$

Where

$$\begin{aligned} g_1(t) &= -\left(\frac{k\lambda_1 V}{1+aV+bV^2} + 2\beta_1 + \gamma \right) + \frac{k\lambda_1 S(1-bV^2)}{(1+aV+bV^2)^2} \frac{V}{I} \\ g_2(t) &= \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V \right) \frac{I}{V} + \frac{I'}{I} - \frac{V'}{V} - \beta_1 - \lambda_2 I - \beta_2 \end{aligned}$$

Rewriting the second and third equation of system (2.2) as follows

$$\begin{aligned} \frac{I'}{I} &= -\frac{k\lambda_1 SV}{(1+aV+bV^2)I} - (\gamma + \beta_1) \\ \frac{V'}{V} &= \lambda_2 \left(\frac{\alpha_2}{\beta_2} - V(t) \right) \frac{I}{V} - \beta_2 \end{aligned}$$

We have

$$g_1(t) \leq -\left(\frac{k\lambda_1 V}{1+aV+bV^2} + 2\beta_1 + \gamma \right) + \frac{k\lambda_1 S}{(1+aV+bV^2)^2} \frac{V}{I}$$

$$g_1(t) < -\beta_1 + \frac{I'}{I} \quad (3.11)$$

$$g_2(t) \leq -\beta_1 + \frac{I'}{I} \quad (3.12)$$

Hence

$$\sup \{ g_1(t), g_2(t) \} \leq -\beta_1 + \frac{I'}{I}$$

From equation (3.10) and Gronwall's inequality¹¹, we obtain

$$L_3(t) \leq L_3(0) I(t) e^{-\beta_1 t} < L_3(0) \frac{\alpha_1}{\mu_1} e^{-\beta_1 t}$$

which implies that $L_3(t) \rightarrow 0$ as $t \rightarrow \infty$. By (3.6), we obtain

$$(W_1(t), W_2(t), W_3(t)) \rightarrow 0 \quad \text{as } t \rightarrow \infty.$$

Hence the linear system (3.5) is asymptotically stable and hence the periodic solution is asymptotically orbitally stable. Therefore, the endemic equilibrium E^* is globally asymptotically stable^{8,9}.

4. Numerical Analysis

We present the numerical simulation using MATLAB, to validate the theoretical results. Figure 1 shows that the disease free equilibrium exists for $R_0 < 1$. Figure 2 indicates that disease becomes endemic for $R_0 > 1$. As the parameters **a** and **b** increase, number of infective individuals decreases. This is shown in Figure 3 and Figure 4, respectively.

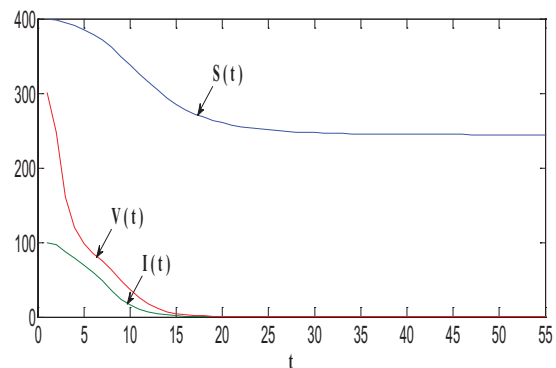


Figure 1. Here $S(0) = 400$, $I(0) = 100$, $V(0) = 300$, $k = 1.1$, $\alpha_1 = 11$, $\alpha_2 = 100$, $\lambda_1 = 0.007$, $\lambda_2 = 0.009$, $\beta_1 = 0.045$, $\beta_2 = 0.61$, $\gamma = 0.12$, $a = 0.02$, $b = 0.01$, $R_0 = 0.401$.

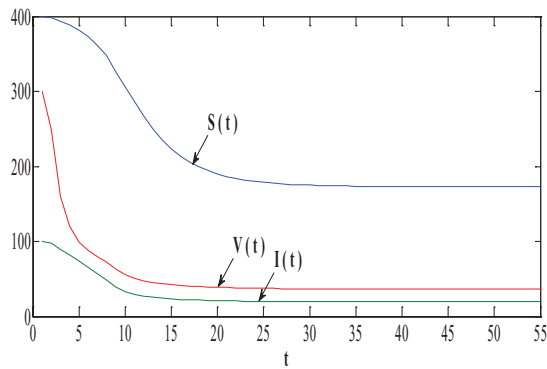


Figure 2. Here $S(0) = 400$, $I(0) = 100$, $V(0) = 300$, $k = 0.016$, $\alpha_1 = 11$, $\alpha_2 = 100$, $\lambda_1 = 0.007$, $\lambda_2 = 0.009$, $\beta_1 = 0.045$, $\beta_2 = 0.61$, $\gamma = 0.12$, $a = 0.02$, $b = 0.01$, $R_0 = 27.59$.

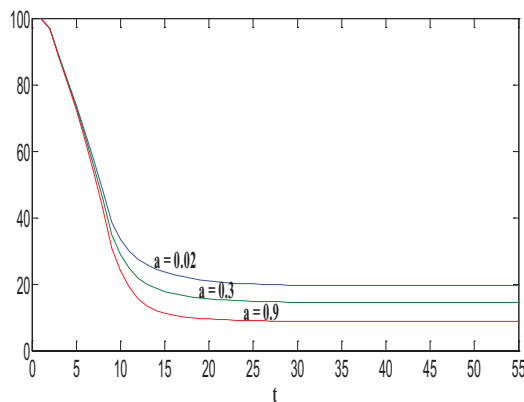


Figure 3. Dependence of I^* on a .

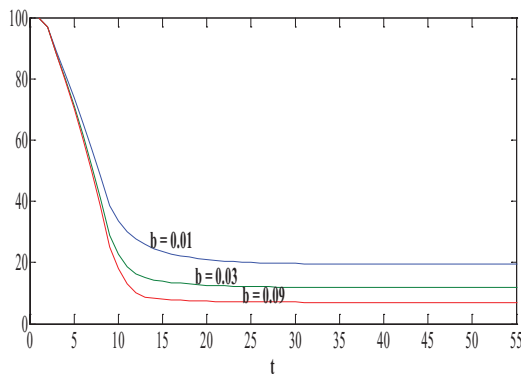


Figure 4. Dependence of I^* on b .

5. Conclusion

We have proposed a vector-host epidemiological model and have studied the dynamical behavior of the model. We established the global asymptotic stability of the

disease free equilibrium for $R_0 \leq 1$ and proved that if $R_0 > 1$ the disease becomes endemic. The expression (2.3) shows that R_0 does not depend on a and b but it is clear from numerical analysis that the number of infective hosts in endemic state I^* decreases as a and b increase. It can be found that from expression (2.6) that the number of infective vectors in endemic state V^* approaches zero as b tends to infinity. The results show that if people are aware of preventive measures such as vaccinations and use of bed nets etc, then the spread of disease can be controlled among the host population. Also, awareness of clean surroundings and use of insecticides helps to reduce the number of vectors and eventually eradicate the disease.

6. References

- Okosun KO, Makinde OD. Optimal control analysis of malaria in the presence of non-linear incidence rate. Appl Comput Math. 2013; 12(1):20–32.
- Ross R. The prevention of malaria. London: Murray; 1911.
- Cai L, Li X. Analysis of a simple vector-host epidemic model with direct transmission. Discrete Dynamics in Nature and Society; 2010.
- Xiao D, Ruan S. Global Analysis of an epidemic model with nonmonotone incidence rate. Math Biosci. 2007; 208:419–29.
- Wei H-M, Li X-Z, Martcheva M. An epidemic model of a vector-borne disease with direct transmission and time delay. Journal of Mathematical Analysis and Applications. 2008; 342(2):895–908.
- Guo S, Li X, Ghosh M. Analysis of a dengue disease model with nonlinear incidence. Discrete Dynamics in Nature and Society; 2013.
- Ozair M, Lashari AA, Jung IH, Okosun KO. Stability analysis and optimal control of a vector-borne disease with nonlinear incidence. Discrete Dynam in Nat and Soc. 2012
- Cai L, Guo S, Li X, Ghosh M. Global dynamics of a dengue epidemic mathematical model. Chaos, Solitons and fractals. 2009; 42:2297–304.
- Cai L, Li X. Global analysis of a vector host epidemic model with nonlinear incidences. Applied Mathematics and Computation. 2010; 217:3531–41.
- Li MY, Muldowney JS. Global stability for the SEIR model in epidemiology. Mathematical Biosciences. 1995; 125:155–64.
- Perko L. Differential Equations and Dynamical Systems. New York: Springer-Verlag; 1996. p. 7.
- Athithan S, Shukla VP, Biradar SR. Voting rule based cellular automata epidemic spread model for *Leptospirosis*. Indian Journal of Science and Technology. 2015; 8(4):337–41.