

An Inventory Model for Decaying Item with Ramp Demand pattern under Inflation and Partial Backlogging

Yogendra Kumar Rajoria^{1*}, S. R. Singh² and Seema Saini¹

¹Department of Mathematics, Graphic Era University, Dehradun, Uttarakhand, India; yogendrarajo@gmail.com, sainiseema1@gmail.com

²Department of Mathematics, C.C.S. University, Meerut, U.P, India

Abstract

The present study proposed a mathematical model for decaying products with ramp type demand function under inflation. Inventory holding cost is an integral term of total cost of inventory organization. Partial backlogging is the decreasing function of waiting time. We explain numerical examples to obtain average total cost per unit time to understand the behavior of inventory system. We also used sensitivity analysis to show effect of changes in total optimal cost per unit time to illustrate the model. We use such type of demand pattern in which seasonal goods come in market, demand of items increases with time and become constant, and decreases to zero or some constant limit. So we use trapezoidal type demand function in place of other demand pattern under inflationary environments. Shortages are permitted which are partially backlogged.

Keywords: Holding Cost, Inflation, Inventory, Ramp Type Demand, Shortages

1. Introduction

Preservation of inventories of decaying products play an important role in an integrated manufacturing process of any management system. Lots of items which are used in daily life start to deteriorate item after sometime. Supplies such as vegetables, fruits having short life period which start deteriorate while held in reserve in stock. Gasoline, alcohol are highly volatile liquid, these liquid will start deteriorates in excess of time through the growth of evaporation¹. Proposed an integrated inventory model for exponentially deteriorated products^{2,3}. Presented an Economic order Quantity model for decaying item for weibull and gamma distribution^{4,5}. Investigated surveys on current growth in constantly decaying products. We considered two types of demand rate: (1) linearly trend demand rate and (2) exponentially varying demand rate. On the other hand, demand rate cannot increases in excess of time. For example, demand of trendy product increases with time for a certain moment in first phase then it become

constant in second phase. This kind of demand is called ramp demand pattern. But in case of trapezoidal demand, demand of second phase decreases with time⁶. Developed mathematical model of deteriorating item for increased demand come after the level demand⁷. Developed a best possible ordering strategy of ramp type inventory for decaying products⁸. Formulated an Economic order quantity model to extend ramp demand pattern with weibull distribution under shortages⁹. Investigated a mathematical model with time unreliable demand, linear replenishment rate under shortages¹⁰. Presented an inventory replenishment strategy for failing products with shortages and partial backlogging¹¹. Proposed a supply chain organization with partial backlogging modeled depend on linear function. However, in the above models, carrying cost per unit time is taken as constant. Inventory decision maker generally has to face the major concern to hold the decaying items. For smooth running of business inventory should be available for whole cycle time in good condition. So inventory holding cost is an integral term

*Author for correspondence

of whole cost of inventory. For better demonstrating such real life situations, carrying cost per unit time should be more general the same as possible¹². Developed a supply chain model for decaying products with cost dependent demand and time changeable holding cost¹³. Developed an integrated inventory model for time dependent order and carrying cost with partial backlogging¹⁴. Developed a most favorable replenishment strategy for life time stock with partial backlogging¹⁵. Developed a best possible ordering strategy with ramp kind of demand function of two level of tool for financing growth¹⁶. Developed a manufacture strategy for ameliorating decaying products with ramp kind of demand¹⁷. Investigated an integrated supply chain model for decaying products, taking into consideration of multivariate expenditure rate with partial backlogging¹⁸. Developed an economics production quantity model with trapezoidal kind of demand in quantity elasticity¹⁹. Developed multi chain integrated inventory model for decaying products with running out date and permissible shortages.

By the observations from the listed research articles above, we have seek out that a lot of requirement of assumption of variable carrying cost per unit time in the inventory models where decaying items are undertaken. Since, this study is based on trapezoidal ramp type demand and inflation. For giving better understandings about the inventory of deteriorating stock by this study, we have considered deterioration governed holding cost per unit time. The rate of holding cost, variable part, handling charge, is exponentially governed by the rate of decaying.

2. Assumptions and Notations

To develop this integrated mathematical model, following assumptions and notations are made:

2.1 Notations

$I(t)$: The level of inventory at any instant $t, t \geq 0$;

S : Highest or beginning level of inventory at the instance $t = 0$ (units);

Q : Ordered size per cycle;

T : Each order of inventory cycle has fixed duration;

t_1 : Inventory approaches zero at any instant;

θ : The constant deterioration rate;

α : The discount rate;

i : The rate of Inflation;

$r(\alpha - i)$: The total concession rate of inflation is constant;

A : The fixed ordering rate per cycle;

c_1 : The price of each deteriorated product;

c_2 : Shortage price per unit back-ordered for each unit time (\$/item/unit time);

c_3 : Opportunity price payable to missing sales (\$/unit);

TAC_1 : The total average price for each unit time under the situation is $t_1 \leq \mu_1$;

TAC_2 : The total average price for each unit time under the situation is $\mu_1 \leq t_1 \leq \mu_2$;

TAC_3 : The total average price for each unit time under the situation is $\mu_2 \leq t_2 \leq T$.

2.2 Assumptions

We required following assumptions to grow inventory model:

- Inventory system deals with single item in stock.
- Lead time is.
- The replenishment rate is instantaneous and infinite rate.
- The failing rate θ ($0 < \theta < 1$), is stable and no item repaired during this interval.
- $D(t)$ is ramp type demand function with respect to moment in time, given by

$$D(t) = \begin{cases} a_1 + b_1 t & t \leq \mu_1, \\ D_0 & \mu_1 \leq t \leq \mu_2 \\ a_2 - b_2 t & \mu_2 \leq t \leq T \leq \frac{a_2}{b_2} \end{cases}$$

Where μ_1 is period of time in which demand function is varying from linearly rising to stable, and μ_2 is period of time in which the demand is varying from stable to the linearly falling.

- Shortages are permitted which are partially backlogged. The backlogging rate $B(t)$ is a waiting time t for next refill, we consider that $B(t) = e^{-\delta t}$, where $\delta \geq 0$, and t is the waiting time.

3. Mathematical Model

We consider trapezoidal type demand rate for inventory model. Inventory level reached maximum at $t = 0$ to t_1 then replenishment occurs at $t = 0$. Shortages are permitted only in intermission (t_1, T) when stock level reaches zero. Demand function is assumed to be partially

backlogged during intermission (t_1, T). All backlogged item is replaced in next replenishment.

$$I'(t) + \theta I(t) = -D(t) \quad 0 < t < t_1 \quad (1)$$

And

$$I'(t) = -D(t)B(t) \quad t_1 < t < T \quad (2)$$

On solving equation (1) and (2) under the boundary condition $I(t_1) = 0$

We discussed only three possible cases which depend on value of t_1, μ_1 and μ_2 .

Case 1 ($0 < t_1 \leq \mu_1$)

Inventory level decreases gradually then falls to zero during time interval $(0, t_1)$, then, from (1) and (2), we have

$$I'(t) + \theta I(t) = -(a_1 + b_1 t), \quad 0 < t < t_1 \quad (3)$$

$$I'(t) = -(a_1 + b_1 t)B(T - t), \quad t_1 < t < \mu_1 \quad (4)$$

$$I'(t) = -D_0 B(T - t), \quad \mu_1 < t < \mu_2 \quad (5)$$

And

$$I'(t) = -(a_2 - b_2 t)B(T - t), \quad \mu_2 < t < T \quad (6)$$

On solving the differential equations (3) to (6) with $I(t_1) = 0$,

The beginning stock level is

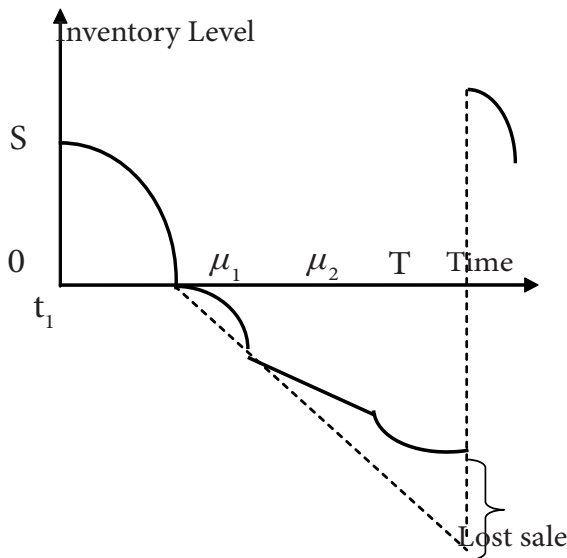


Figure 1. A Graphical depiction of stock level in cycle.

$$S = I(0) = \left(\frac{a_1 - b_1}{\theta} \right) (e^{\theta t_1} - 1) + \frac{b_1 t_1 e^{\theta t_1}}{\theta}.$$

Replenishment takes place at time $t = 0$ and T , demand and deterioration both consumed the item which is replenished during t_1 . The current value of price of material is given by

$$MC = c_1 S + c_1 e^{-rt} \left[\int_{t_1}^{\mu_1} (a_1 + b_1 t) dt + \int_{\mu_1}^{\mu_2} D_0 dt + \int_{\mu_2}^T (a_2 - b_2 t) dt \right]$$

The current value of carrying price per cycle is

$$HC = \int_0^{t_1} RI(t) e^{-rt} dt \quad (7)$$

The current value of shortage price per cycle is

$$SC = - \int_{t_1}^{\mu_1} c_2 I(t) e^{-rt} dt - \int_{\mu_1}^{\mu_2} c_2 I(t) e^{-rt} dt - \int_{\mu_2}^T c_2 I(t) e^{-rt} dt$$

The current value of opportunity charge due to missing sale is

$$OPC = \left[c_3 \int_{t_1}^{\mu_1} e^{-rt} (1 - B(t)) (a_1 + b_1 t) dt + c_3 \int_{\mu_1}^{\mu_2} e^{-rt} (1 - B(t)) D_0 dt + c_3 \int_{\mu_2}^T e^{-rt} (1 - B(t)) (a_2 - b_2 t) dt \right]$$

Then, the total average cost for condition ($t_1 \leq \mu_1$) is computed by $TAC_1(t_1) = \frac{1}{T} [A + MC + HC + SC + OPC]$ (8)

Total backordering cost for this cycle is

$$B_1 = a_1 (\mu_1 - t_1^*) + \frac{b_1 (\mu_1^2 - t_1^{*2})}{2} + D_0 (\mu_2 - \mu_1) + a_2 (T - \mu_2) - \frac{b_2 (T^2 - \mu_2^2)}{2}$$

Therefore, Q^* is an order quantity, then $Q^* = S^* + B_1$, where S^* is best possible price of S .

Case 2 ($\mu_1 \leq t_1 \leq \mu_2$)

Inventory level decreases gradually then falls to zero during time interval $(0, t_1)$ then from (1) and (2), we have

$$I'(t) + \theta I(t) = -(a_1 + b_1 t), \quad 0 < t < \mu_1 \quad (9)$$

$$I'(t) + \theta I(t) = -D_0, \quad \mu_1 < t < t_1 \quad (10)$$

$$I'(t) = -D_0B(T-t), \quad t_1 < t < \mu_2 \quad (11)$$

And

$$I'(t) = -(a_2 - b_2t)B(T-t), \quad \mu_2 < t < T \quad (12)$$

Solving the differential equations (9)–(12) with $I(t_1) = 0$

The beginning stock level can be calculated as

$$S = I(0) = \frac{D_0e^{\theta t_1}}{\theta} - \frac{b_1e^{\theta t_1}}{\theta^2} + \frac{b_1}{\theta^2} - \frac{a_1}{\theta}$$

Replenishment takes place at time $t = 0$ and T , demand and deterioration both consumed the item which is replenished during. The current value of price of material is given by

$$MC = c_1S + c_1e^{-rT} \left[\int_{t_1}^{\mu_2} D_0 dt + \int_{\mu_2}^T (a_2 - b_2t) dt \right] \quad (13)$$

The current value of carrying price per cycle is

$$HC = \int_0^{\mu_1} RI(t)e^{-rt} dt + \int_{\mu_2}^{t_1} RI(t)e^{-rt} dt \quad (14)$$

The essential condition for in $TAC_1(t_1)$ in (8) to be minimized

$$\frac{dTAC_1(t_1)}{dt_1} = 0$$

The root of (8) will be the optimal solution t_1^* for $t_1^* \in (0, \mu_1)$ if at $t_1 = t_1^*$

$$\frac{d^2TAC_1(t_1)}{dt_1^2} > 0$$

$$SC = - \int_{t_1}^{\mu_2} c_2I(t)e^{-rt} dt - \int_{\mu_2}^T c_2I(t)e^{-rt} dt \quad (15)$$

The current value of opportunity expenditure due to shortages is

$$HC = \int_0^{\mu_1} RI(t)e^{-rt} dt + \int_{\mu_1}^{\mu_2} RI(t)e^{-rt} dt + \int_{\mu_2}^T RI(t)e^{-rt} dt$$

The stock-out cost for this order cycle is

$$SC = - \int_{t_1}^T c_2I(t)e^{-rt} dt \quad (23)$$

The cost of opportunity during to shortage is

$$OPC = c_3 \left[\int_{t_1}^T e^{-rt} (1 - B(t))(a_2 - b_2t) dt \right] \quad (24)$$

$$OPC = c_3 \left[\int_{t_1}^{\mu_2} e^{-rt} (1 - B(t))D_0 dt + \int_{\mu_2}^T e^{-rt} (1 - B(t))(a_2 - b_2t) dt \right]$$

Then, total average cost under the condition $(\mu_1 \leq t_1 \leq \mu_2)$ computed as

$$TAC_2(t_1) = \frac{1}{T} [A + MC + HC + SC + OPC] \quad (16)$$

The total backordering cost for this cycle is given by

$$B_2 = D_0(\mu_2 - t_1^*) + a_2(T - \mu_2) - \frac{b_2(T^2 - \mu_2^2)}{2} \quad (17)$$

Therefore, Q^* is an order size, then $Q^* = S^* + B_2$,

Where S^* is best possible price of S .

Case 3 $\mu_2 \leq t_2 \leq T$

Stock level decreases steadily then decreases to zero during moment in time period $(0, t_1)$ then from (1) and (2), we have

$$I'(t) + \theta I(t) = -(a_1 + b_1t), \quad 0 < t < \mu_1 \quad (18)$$

$$I'(t) + \theta I(t) = -D_0, \quad \mu_1 < t < \mu_2 \quad (19)$$

$$I'(t) + \theta I(t) = -(a_2 - b_2t), \quad \mu_2 < t < t_1 \quad (20)$$

And

$$I'(t) = -(a_2 - b_2t)B(T-t), \quad t_1 < t < T \quad (21)$$

Solving the differential equations (18)–(21) with $I(t_1) = 0$,

The initial stock level can be calculated as

$$S = I(0) = \frac{\theta(a_2 - b_2t_1) + b_2}{\theta^2} e^{\theta t_1} - \frac{b_1e^{\theta t_1}}{\theta^2} - \frac{b_2e^{\theta \mu_2}}{\theta^2} + \frac{b_1 - a_1\theta}{\theta^2}$$

Replenishment take place at time $t = 0$ and T , demand and deterioration both consumed the item which is replenished during. The current value of price of material is given by

$$MC = c_1S + c_1e^{-rT} \left[\int_{\mu_2}^T (a_2 - b_2t) dt \right] \quad (22)$$

The current value of carrying price per cycle is

$$TAC_3(t_1) = \frac{1}{T} [A + MC + HC + SC + OPC] \quad (25)$$

Back-order cost can be calculated at the end of order cycle, which is given by

$$B_3 = a_2(T - \mu_2) - \frac{b_2(T^2 - \mu_2^2)}{2} \tag{26}$$

Therefore, Q^* is an order size, then $Q^* = S^* + B_3$, where S^* is best possible price of S .

4. Numerical Analysis

Based on the previous studies, we consider the following data in appropriate units:

- Example 1:** For Case 1 we deduce an inventory structure which satisfied our assumptions. The input data of parameters are taken randomly as $A = 30, T = 3, \mu_1 = 1.2, \mu_2 = 2.7, a_1 = 3, b_1 = 0.4, a_2 = 8, b_2 = 3, \theta = 0.3, R = 0.5, c_2 = 0.6, \delta = 0.11, c_1 = 0.9, r = 0.1, c_3 = 1, D_0 = 9$. By using MATHEMATICA 8.0, the minimum total average cost per unit time $TAC_1(t_1^*)$, along with the optimal value of t_1^* is calculated for case 1. The optimal total maximum quantity (S^*) is also calculated in this case. Therefore, the optimal policies for case $(0 < t_1 \leq \mu_1)$ under assumed conditions are $t_1^* = 0.79948, S^* = 8.60231$, and $TAC_1(t_1^*) = 6.35431$ in their appropriate units.
- Example 2:** For Case 2 we deduce an inventory structure which satisfied our assumptions. The input data of parameters are taken randomly as $A = 20, T = 8, \mu_1 = 3, \mu_2 = 7, a_1 = 1, b_1 = 0.5, a_2 = 10, \theta = 0.39, R = 0.1, c_2 = 0.01, \delta = 0.25, b_2 = 1, c_3 = 1, D_0 = 10, c_1 = 0.3, r = 0.1$. By using MATHEMATICA 8.0, the least amount of total average cost / unit time $TAC_2(t_1^*)$, along with the best possible price of t_1^* is calculated for case the best possible total maximum quantity (S^*) is also calculated in this case. Therefore, the best possible policies for case $(\mu_1 \leq t_1 \leq \mu_2)$ under assumed conditions are $t_1^* = 4.60231, S^* = 22.022$ and $TAC_2(t_1^*) = 16.8339$ in their appropriate units.
- Example 3:** For Case 3 we deduce an inventory structure which satisfied our assumptions. The input data of parameters are taken randomly as $A = 20, T = 8, \mu_1 = 3, \mu_2 = 8, a_1 = 3, b_1 = 0.5, a_2 = 10, \theta = 0.3, R = 2, c_2 = 20, \delta = 0.11, c_3 = 1, D_0 = 10, c_1 = 3, c_3 = 1, D_0 = 10, c_1 = 3, r = 0.21$. By using MATHEMATICA 8.0, the least amount of average total cost per unit time $TAC_3(t_1^*)$, along with the best possible price of t_1^* is calculated for case 3. The best possible total maximum size (S^*) is also calculated in this

case. Therefore, the optimal policies for case $(\mu_2 \leq t_1 \leq T)$ under assumed conditions are $t_1^* = 7.65196, S^* = 25.7463$ and $TAC_3(t_1^*) = 38.698$ in their appropriate units.

5. Sensitivity Analysis

In this part, we discussed the changes in the best possible price of the total average cost per unit time, and the maximum size per cycle with regard to changes in a few parameters of model.

We performed the sensitivity analysis for both the above cases by varying the cost of each of the parameters by $\pm 5\%$ and $\pm 10\%$, choosing one parameter at instance and fixing the remaining parameters unaltered. Example 1, 2 and 3 are used separately.

5.1 Sensitivity Analysis for Case 1

In case $(0 < t_1 \leq \mu_1)$, to discuss the effect of changes of model parameters R, H, c_1, c_2 , and c_3 on the best possible price of the total average cost ($TAC_1(t_1^*)$), the shortage point (t_1^*), and the total maximum quantity per cycle S^* for case 1, the different values of these parameter according to $\pm 5\%$ and $\pm 10\%$ change in each have taken and its effect on $TAC_1(t_1^*), t_1^*$ and S^* are existing in the following Table 1.

Table 1. Sensitivity Analysis for Case 1

Parameter	t_1^*	S^*	$TAC_1(t_1^*)$
$R = 0.5$	0.78546	8.600321	6.33171
	0.79300	8.601313	6.31301
	0.80598	8.603311	6.29561
	0.81250	8.604315	6.27690
$c_1 = 0.9$	0.498565	0.631188	5.69419
	0.489222	0.616639	5.52419
	0.470740	0.588197	5.18456
	0.461600	0.574295	5.01493
$c_2 = 0.6$	0.285517	0.326884	6.59843
	0.378731	0.453125	5.98127
	0.590237	0.780218	4.71781
	0.710861	0.994517	4.07245
$c_3 = 1$	0.668665	0.917111	3.52997
	0.575144	0.754885	4.44697
	0.382987	0.459139	6.25105
	0.284169	0.325133	7.13616

6. Conclusion

Demand rate of product increases with time then it become stabilizes; finally it decreases to zero or constant. Inventory system has several replenishment and fixed length of order cycle. We used such type of demand function when seasonal products come in market. Shortages allowed which are decreasing function of time. The numerical examples have been given in this study to give illustration of the assumptions. The proper sensitivity analysis for each case carried out for give the picture of proposed model's behavior. Since this study deals with decaying item with trapezoidal demand and inflation, inventory managers may use this study in wide extend of decision making in inventory controlling. There are several hopeful areas for further research. In this study, there is a trapezoidal demand rate with linear pattern in every interval is considered. In the future version of the present study, this mathematical model can be improved by allowing for exponentially increasing demand in every interval of trapezoidal demand function. Another area for further research is permissible delay assumption in the present study.

7. References

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