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New Zero-stable Block Method for Direct Solution of Fourth Order Ordinary Differential Equations

J. O. Kuboye* and Z. Omar

School of Quantitative Sciences, College of Art and Sciences, Universiti Utara Malaysia, Sintok, 06010, Malaysia; Kubbysholly2007@yahoo.com

Abstract

A zero-stable block method for solving fourth order initial value problems of ordinary differential equations directly is proposed. Interpolation and collocation techniques are employed in developing the method. Power series approximate is used as a basis function where its fourth derivative is collocated at all the grid points within the specified interval. Furthermore, the properties of the method developed, that is, zero-stability, consistency, convergence, order and error constant are established. The new method performs better than the existing methods in terms of accuracy.

Keywords: Block Method, Collocation, Direct Solution, Grid Points, Interpolation, Ordinary Differential Equations

1. Introduction

In this paper, a fourth order ordinary differential equations of the form

$$y^{(4)} = f(x, y, y', y'', y''') \quad x \in [x_0, b]$$

with initial conditions $y^{(m)}(x_0) = y_m,\ m = 0(1)3$ is considered.

Conventionally, (1) is transformed into an equivalent system of first order ordinary differential equations and thereafter suitable numerical method would be applied to solve the resultant system. This technique is widely discussed by authors [5,7,11,12] amongst others. In spite of the success recorded on this approach, a lot of setbacks were also discovered such as computational burden that usually affects the accuracy of the results and the computation time.

To cater for the setbacks mentioned above, several researchers [2–3,6,13,15–18] to mention a few, developed methods for solving higher order directly without going through the process of reduction. It was found that this method is better in terms of accuracy than when the differential equation is reduced to system of first order ordinary differential equations.

Numerical schemes which include the predictor-corrector method and block method have been proposed by many scholars. It is observed that predictors are in reducing order of accuracy which always affects the efficiency of the predictor-corrector method. In addition, the method requires much human effort and computer time[9].

In order to overcome the weaknesses in predictor-corrector method, block method was introduced by Milne in 1953. It was formerly used as a predictor for predictor-corrector algorithm and later adopted as a full method[1,4]. Scholars such as[2,10,13,19] adopted a block method for direct solution of higher order ordinary differential equations whereby it accuracy was confirmed better than predictor-corrector method.

* Author for correspondence

In this study, a zero-stable block method for step-length of six is proposed. The method employs collocation and interpolation strategy.

2. Derivation of the Method

A power series of the form $y(x)=\sum_{j=0}^{k+4} a_j x^j$ .....(2) is considered as an approximate solution to equation(1)for k=6. Differentiate (2) four times gives

$$y^{(4)}(x)=\sum_{j=4}^{k+4} j(j-1)(j-2)(j-3)a_j x^{j-4}$$

Equation (2) is interpolated at the selected grid points $x=x_{n+i} i=(k-4)(1)(k-2)$ and equation (3) is collocated at the points. $x=x_{n+i} i=0(1)k$ This gives a system of non-linear equation of the form

$$\sum_{j=0}^{k+4} a_j x^j = y_{n+i}$$

$$\sum_{j=4}^{k+4} j(j-1)(j-2)(j-3)a_j x^{j-4} = f_{n+i}$$

Solving (4) for the unknowns constants a's and substituting into (2) gives a continuous linear multistep method in the form

$$y(z)=\sum_{j=1}^{k-2}\alpha_j(z)y_{n+j}+h^4\sum_{j=0}^{k}\beta_j(z)f_{n+j}$$

Where

$$z=\frac{x-x_{n+k-1}}{h}$$

$$\alpha_1(z)=-1-\frac{11z}{6}-z^2-\frac{z^3}{6}$$

$$\alpha_2(z)=4+7z+\frac{7z^2}{2}+\frac{z^3}{2}$$

$$\alpha_3(z)=-6-\frac{19z}{2}-4z^2-\frac{z^3}{2}$$

$$\alpha_4(z)=4+\frac{13z}{3}+\frac{3z^2}{2}+\frac{z^3}{6}$$

$$\beta_0(z)=\frac{1}{3628800}(1200+1984z+2088z^2+1955z^3$$

$$\beta_1(z)=\frac{1}{3628800}(-12240-19740z-17898z^2-15080z^3+7560z^5+2604z^6-720z^7-540z^8-100z^9-6z^{10})$$

$$\beta_2(z)=\frac{1}{3628800}(642960+1164900z+655740z^2+149815z^3-25200z^5-7980z^6+2610z^7+1665z^8+275z^9+15z^{10})$$

$$\beta_3(z)=\frac{1}{3628800}(2364960+4576600z+2655060z^2+409560z^3+50400z^5+13160z^6-5760z^7-2760z^8-400z^9-20z^{10})$$

$$\beta_4(z)=\frac{1}{3628800}(642960+1816500z+1851000z^2+740705z^3-75600z^5-7140z^6+7470z^7+2565z^8+325z^9+15z^{10})$$

$$\beta_5(z)=\frac{1}{3628800}(-12240+18852z+150822z^2+231920z^3+151200z^4+38808z^5-4116z^6-5040z^7-1260z^8-140z^9-6z^{10})$$

$$\beta_6(z)=\frac{1}{3628800}(1200+940z-4812z^2-6875z^3+5040z^5+3836z^6+1350z^7+255z^8+25z^9+z^{10})$$

Evaluating (5) at $x=x_{n+i}, i=0,5$and 6, the first, second and third derivative at$x=x_{n+i}, i=0(1)6$. This gives the discrete schemes and its derivatives. Therefore using the matrix inversion, this produces the block of the form

$$A^0Y_N = A'Y_{N-1}+hA''Y'_{N-1}+h^2B'Y''_{N-1}+h^3B''Y'''_{N-1}+h^4(E^0F_N+E'F_{N-1})$$

where

$$Y_N=[y_{n+1},y_{n+2}..,y_{n+k}]^T, Y_{N-1}=[y_{n-k+1},y_{n-k+2}...,y_n]^T$$
$$Y'_{N-1}=[y'_{n-k+1},y'_{n-k+2}..,y'_n]^T, Y''_{N-1}=[y''_{n-k+1},y''_{n-k+2}..,y''_n]$$
$$Y'''_{N-1}=[y'''_{n-k+1},y'''_{n-k+2}..,y'''_n]^T, F_{N-1}=[f_{n-k+1},f_{n-k+2}...,f_n]^T,$$
$$F_N=[f_{n+1},f_{n+2}..,f_{n+k}]^T$$

$$A^0=\begin{pmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix},A'=\begin{pmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{pmatrix}\quad A''=\begin{pmatrix}0&0&0&0&0&1\\0&0&0&0&0&2\\0&0&0&0&0&3\\0&0&0&0&0&4\\0&0&0&0&0&5\\0&0&0&0&0&6\end{pmatrix},B'=\begin{pmatrix}0&0&0&0&0&\frac{1}{2}\\0&0&0&0&0&2\\0&0&0&0&0&\frac{9}{2}\\0&0&0&0&0&8\\0&0&0&0&0&\frac{25}{2}\\0&0&0&0&0&18\end{pmatrix}$$

$$B''=\begin{pmatrix}0&0&0&0&0&\frac{1}{6}\\0&0&0&0&0&\frac{4}{3}\\0&0&0&0&0&\frac{9}{2}\\0&0&0&0&0&\frac{32}{3}\\0&0&0&0&0&\frac{125}{6}\\0&0&0&0&0&36\end{pmatrix}$$

$$E^0=\begin{pmatrix}\frac{74}{2397}&\frac{-157}{4947}&\frac{181}{6749}&\frac{-196}{13303}&\frac{110}{23653}&\frac{-17}{26556}\\\frac{1135}{1832}&\frac{-199}{405}&\frac{388}{945}&\frac{-127}{567}&\frac{998}{14175}&\frac{-137}{14175}\\\frac{1447}{473}&\frac{-721}{409}&\frac{261}{160}&\frac{-196}{219}&\frac{3159}{11200}&\frac{-571}{14727}\\\frac{5050}{573}&\frac{-667}{187}&\frac{1361}{314}&\frac{-928}{405}&\frac{2048}{2835}&\frac{-415}{4178}\\\frac{11431}{591}&\frac{-1743}{320}&\frac{7039}{717}&\frac{-5539}{1231}&\frac{3661}{2489}&\frac{-358}{1769}\\\frac{6318}{175}&\frac{-243}{35}&\frac{684}{35}&\frac{-243}{35}&\frac{486}{175}&\frac{-9}{25}\end{pmatrix}$$

$$E'=\begin{pmatrix}0&0&0&0&0&\frac{93}{3518}\\0&0&0&0&0&\frac{513}{1762}\\0&0&0&0&0&\frac{721}{656}\\0&0&0&0&0&\frac{997}{362}\\0&0&0&0&0&\frac{1607}{289}\\0&0&0&0&0&\frac{1719}{175}\end{pmatrix}$$

The first, second and third derivatives of (6) give

$$\begin{pmatrix}y'_{n+1}\\y'_{n+2}\\y'_{n+3}\\y'_{n+4}\\y'_{n+5}\\y'_{n+6}\end{pmatrix}=W\begin{pmatrix}y'_n\\hy''_n\\\frac{h^2}{2}y'''_n\end{pmatrix}+h^3U\begin{pmatrix}f_{n+6}\\f_{n+5}\\f_{n+4}\\f_{n+3}\\f_{n+2}\\f_{n+1}\\f_n\end{pmatrix}$$

$$W=\begin{pmatrix}1&1&1\\1&2&4\\1&3&9\\1&4&16\\1&5&25\\1&6&36\end{pmatrix}$$

$$\begin{pmatrix}y''_{n+1}\\y''_{n+2}\\y''_{n+3}\\y''_{n+4}\\y''_{n+5}\\y''_{n+6}\end{pmatrix}=\begin{pmatrix}1&1\\1&2\\1&3\\1&4\\1&5\\1&6\end{pmatrix}\begin{pmatrix}y''_n\\hy'''_n\end{pmatrix}+h^2L\begin{pmatrix}f_{n+6}\\f_{n+5}\\f_{n+4}\\f_{n+3}\\f_{n+2}\\f_{n+1}\\f_n\end{pmatrix}.\ \text{Where}$$

$$U=\begin{pmatrix}\frac{-9809}{3628800}&\frac{71364}{3628800}&\frac{-226605}{3628800}&\frac{414160}{3628800}&\frac{-494715}{3628800}&\frac{506604}{3628800}&\frac{343801}{3628800}\\\frac{-491}{28350}&\frac{3576}{28350}&\frac{-11370}{28350}&\frac{20800}{28350}&\frac{-24465}{28350}&\frac{35976}{28350}&\frac{13774}{28350}\\\frac{-1917}{44800}&\frac{13932}{44800}&\frac{-44145}{44800}&\frac{80640}{44800}&\frac{-72495}{44800}&\frac{172692}{44800}&\frac{52893}{44800}\\\frac{-2272}{28350}&\frac{16512}{28350}&\frac{-52080}{28350}&\frac{108800}{28350}&\frac{-54240}{28350}&\frac{223872}{28350}&\frac{61808}{28350}\\\frac{-18625}{145152}&\frac{136500}{145152}&\frac{-358125}{145152}&\frac{1070000}{145152}&\frac{-256875}{145152}&\frac{1945500}{145152}&\frac{505625}{145152}\\\frac{-126}{700}&\frac{1296}{700}&\frac{-1620}{700}&\frac{8640}{700}&\frac{-810}{700}&\frac{14256}{700}&\frac{3564}{700}\end{pmatrix}$$

$$L=\begin{pmatrix}\frac{-995}{120960}&\frac{-7254}{120960}&\frac{-23109}{120960}&\frac{42484}{120960}&\frac{-51453}{120960}&\frac{57750}{120960}&\frac{28549}{120960}\\\frac{-2432}{120960}&\frac{17664}{120960}&\frac{-55872}{120960}&\frac{100864}{120960}&\frac{-107520}{120960}&\frac{223488}{120960}&\frac{65728}{120960}\\\frac{-141}{4480}&\frac{1026}{4480}&\frac{-3267}{4480}&\frac{6300}{4480}&\frac{-2403}{4480}&\frac{14850}{4480}&\frac{3795}{4480}\\\frac{-40}{945}&\frac{288}{945}&\frac{-840}{945}&\frac{2624}{945}&\frac{-72}{945}&\frac{4512}{945}&\frac{1088}{945}\\\frac{-1375}{24192}&\frac{11550}{24192}&\frac{-5625}{24192}&\frac{102500}{24192}&\frac{9375}{24192}&\frac{150750}{24192}&\frac{35225}{24192}\\0&\frac{216}{140}&\frac{54}{140}&\frac{816}{140}&\frac{108}{140}&\frac{1080}{140}&\frac{246}{140}\end{pmatrix}$$

$$\begin{pmatrix} y'''_{n+1} \\ y'''_{n+2} \\ y'''_{n+3} \\ y'''_{n+4} \\ y'''_{n+5} \\ y'''_{n+6} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} y'''_n + hV \begin{pmatrix} f_{n+6} \\ f_{n+5} \\ f_{n+4} \\ f_{n+3} \\ f_{n+2} \\ f_{n+1} \\ f_n \end{pmatrix} where$$

$$V = \begin{pmatrix} \frac{-863}{60480} & \frac{6312}{60480} & \frac{-20211}{60480} & \frac{37504}{60480} & \frac{-46461}{60480} & \frac{65112}{60480} & \frac{19087}{60480} \\ \frac{-37}{3780} & \frac{264}{3780} & \frac{-807}{3780} & \frac{1328}{3780} & \frac{33}{3780} & \frac{5640}{3780} & \frac{1139}{3780} \\ \frac{-29}{2240} & \frac{216}{2240} & \frac{-729}{2240} & \frac{2176}{2240} & \frac{1161}{2240} & \frac{3240}{2240} & \frac{685}{2240} \\ \frac{-16}{1890} & \frac{96}{1890} & \frac{348}{1890} & \frac{3008}{1890} & \frac{768}{1890} & \frac{2784}{1890} & \frac{572}{1890} \\ \frac{-275}{12096} & \frac{5640}{12096} & \frac{11625}{12096} & \frac{16000}{12096} & \frac{6375}{12096} & \frac{17400}{12096} & \frac{3715}{12096} \\ \frac{41}{140} & \frac{216}{140} & \frac{27}{140} & \frac{272}{140} & \frac{27}{140} & \frac{216}{140} & \frac{41}{140} \end{pmatrix}$$

3. Analysis of the Properties of the Method

3.1 Order of the Method

Our method (6) has a uniform order $[7,7,7,7,7,7]^T$ together with error constants $[\frac{39}{74183}, \frac{109}{13912}, \frac{243}{7700}, \frac{361}{4457}, \frac{430}{2599}, \frac{8}{275}]^T$. This is done by the method proposed by the author [11].

3.2 Zero Stability

The method (6) is said to be zero-stable if no root of the first characteristic polynomial $\rho(r)$ is having a modulus greater than one and every root of modulus one is simple.

That is, $\rho(r) = \det[rA^0 - A'] = 0.$

A^0 and A^1 are the coefficients of $y_{n+i}, i = 1(1)6$ and y_n in (6). This is demonstrated below

$$\det[rA^{(0)} - A^{(1)}] = \left\| \begin{pmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r & 0 \\ 0 & 0 & 0 & 0 & 0 & r \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\| = 0$$

Hence the roots of the above matrix are 0, 0, 0, 0, 0 with max r=1. Therefore, by the above definition, our method is zero stable.

3.3 Consistency

A method is consistent if the order is greater than one. Hence our method is consistent. Hence the method is convergent.

3.4 Numerical Results

The following problems are considered for the purpose of testing our method

1. $y^{iv} = x,\ y(0)=0, y'(0)=1, y''(0)=0,\ y'''(0)=0,\ h=0.1$

Exact solution: $y(x) = \dfrac{x^5}{120} + x$

The problem above was solved by author[13] using the developed block method of step-length k=6 with h=0.1 for special fourth order ordinary differential equations. The same problem is solved by our method and we compared our result with their result as shown in Table 1 (Refer to the Appendix).

2. $y^{iv} + y'' = 0 \quad 0 \le x \le \dfrac{\pi}{2},\ y(0)=0,\ y'(0) = \dfrac{-1.1}{72-50\pi}$

$y''(0) = \dfrac{1}{144-100\pi}, y'''(0) = \dfrac{1.2}{144-100\pi}$

Exact solution: $y(x) = \dfrac{1 - x - \cos x - 1.2\sin x}{144-100\pi}$

This problem above was solved by[2] where a 5-step bock method was developed with h=0.01. Our method is used to solve the same problem and our result is compared with their result. This is shown in Table 2 (Refer to the Appendix).

3. $y^{iv} = (y')^2 - y(y'') - 4x^2 + e^x(1-4x+x^2) \quad 0 \le x \le 1$

$y(0)=1,\ y'(0)=1,\quad y''(0)=3,\ y'''(0)=1$

Exact solution: $x^2 + e^x$

The above problem was also solved by the scholar[2] using a 5-step block method with h=0.01. We also apply our method to the same problem and compared our result with their result. This is shown is Table 3 (Refer to the Appendix).

4. Conclusion

A zero-stable block method has been developed in this paper. The new method is used to solve fourth order initial value problems of ordinary differential equations and the results generated are compared with those in Tables 1, 2 and 3. The obtained results are found far better than the existing methods in terms of accuracy.

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Appendix

Table 1. Comparison of the results of the new method with the results[13]

X	Exact solution	Computed solution	Error in the method[13] K=6	Error in new method, K=6
0.1	0.100000083333333340	0.100000083333333340	7.0000E-10	0.000000E+00
0.2	0.200002666666666690	0.200002666666666690	8.9999E-10	0.000000E+00
0.3	0.300020250000000040	0.300020250000000040	2.0999E-10	0.000000E+00
0.4	0.400085333333333350	0.400085333333333350	5.1000E-09	0.000000E+00
0.5	0.500260416666666650	0.500260416666666650	7.7999E-09	0.000000E+00
0.6	0.600648000000000070	0.600648000000000070	1.1800E-08	0.000000E+00
0.7	0.701400583333333440	0.701400583333333440	1.2400E-08	0.000000E+00
0.8	0.802730666666666700	0.802730666666666810	1.4100E-08	1.110223E-16
0.9	0.904920750000000160	0.904920750000000050	1.8800E-08	1.110223E-16
1.0	1.008333333333333300	1.008333333333333300	2.6000E-08	0.000000E+00

Table 2. Comparison of the results of the new method with the method[2]

X	Exact solution y(x)	Computed solution	Error in method[2] k=5	Error in new method K=6
0.01	0.000128995622844037	0.000128995622844037	6.5052E-19	5.421011E-20
0.02	0.000257396543210136	0.000257396543210136	1.3010E-18	5.421011E-20
0.03	0.000385195797911474	0.000385195797911474	4.7704E-18	2.710505E-19
0.04	0.000512386483927295	0.000512386483927295	1.7347E-17	1.084202E-19
0.05	0.000638961759093202	0.000638961759093201	4.3368E-17	3.252607E-19
0.06	0.000764914842785370	0.000764914842785370	9.5409E-17	3.252607E-19
0.07	0.000890239016598605	0.000890239016598605	1.8127E-16	0.00000E+00
0.08	0.001014927625018177	0.001014927625018175	3.1571E-16	1.734723E-18
0.09	0.001138974076085363	0.001138974076085358	5.1868E-16	4.336809E-18
0.10	0.001262371842056641	0.001262371842056632	8.0491E-16	8.456777E-18

Table 3. Comparison of the results of the new method with the method[2]

X	Exact solution y(x)	Computed solution	Error in method[2] k=5	Error in new method K=6
0.01	1.010150167084167900	1.010150167084168200	9.0460E-13	2.220446E-16
0.02	1.020601340026755700	1.020601340026755700	2.1516E-12	0.000000E+00
0.03	1.031354533953516800	1.031354533953516800	3.7549E-12	0.000000E+00
0.04	1.042410774192388300	1.042410774192388300	5.6885E-12	0.000000E+00
0.05	1.053771096376024100	1.053771096376024100	7.8819E-12	0.000000E+00
0.06	1.065436546545359700	1.065436546545359700	1.0212E-11	0.000000E+00
0.07	1.077408181254216400	1.077408181254226900	1.2497E-11	1.043610E-14
0.08	1.089687067674958600	1.089687067674993200	1.4486E-11	3.463896E-14
0.09	1.102274283705210400	1.102274283705289200	1.5849E-11	7.882583E-14
0.10	1.115170918075647700	1.115170918075798300	1.6159E-11	1.505462E-13

Appendix

Table 1. Comparison of the results of the new method with the results[13]

X	Exact solution	Computed solution	Error in the method[13] K=6	Error in new method, K=6
0.1	0.100000083333333340	0.100000083333333340	7.0000E-10	0.000000E+00
0.2	0.200002666666666690	0.200002666666666690	8.9999E-10	0.000000E+00
0.3	0.300020250000000040	0.300020250000000040	2.0999E-10	0.000000E+00
0.4	0.400085333333333350	0.400085333333333350	5.1000E-09	0.000000E+00
0.5	0.500260416666666650	0.500260416666666650	7.7999E-09	0.000000E+00
0.6	0.600648000000000070	0.600648000000000070	1.1800E-08	0.000000E+00
0.7	0.701400583333333440	0.701400583333333440	1.2400E-08	0.000000E+00
0.8	0.802730666666666700	0.802730666666666810	1.4100E-08	1.110223E-16
0.9	0.904920750000000160	0.904920750000000050	1.8800E-08	1.110223E-16
1.0	1.008333333333333300	1.008333333333333300	2.6000E-08	0.000000E+00

Table 2. Comparison of the results of the new method with the method[2]

X	Exact solution y(x)	Computed solution	Error in method[2] k=5	Error in new method K=6
0.01	0.000128995622844037	0.000128995622844037	6.5052E-19	5.421011E-20
0.02	0.000257396543210136	0.000257396543210136	1.3010E-18	5.421011E-20
0.03	0.000385195797911474	0.000385195797911474	4.7704E-18	2.710505E-19
0.04	0.000512386483927295	0.000512386483927295	1.7347E-17	1.084202E-19
0.05	0.000638961759093202	0.000638961759093201	4.3368E-17	3.252607E-19
0.06	0.000764914842785370	0.000764914842785370	9.5409E-17	3.252607E-19
0.07	0.000890239016598605	0.000890239016598605	1.8127E-16	0.00000E+00
0.08	0.001014927625018177	0.001014927625018175	3.1571E-16	1.734723E-18
0.09	0.001138974076085363	0.001138974076085358	5.1868E-16	4.336809E-18
0.10	0.001262371842056641	0.001262371842056632	8.0491E-16	8.456777E-18

Table 3. Comparison of the results of the new method with the method[2]

X	Exact solution y(x)	Computed solution	Error in method[2] k=5	Error in new method K=6
0.01	1.010150167084167900	1.010150167084168200	9.0460E-13	2.220446E-16
0.02	1.020601340026755700	1.020601340026755700	2.1516E-12	0.000000E+00
0.03	1.031354533953516800	1.031354533953516800	3.7549E-12	0.000000E+00
0.04	1.042410774192388300	1.042410774192388300	5.6885E-12	0.000000E+00
0.05	1.053771096376024100	1.053771096376024100	7.8819E-12	0.000000E+00
0.06	1.065436546545359700	1.065436546545359700	1.0212E-11	0.000000E+00
0.07	1.077408181254216400	1.077408181254226900	1.2497E-11	1.043610E-14
0.08	1.089687067674958600	1.089687067674993200	1.4486E-11	3.463896E-14
0.09	1.102274283705210400	1.102274283705289200	1.5849E-11	7.882583E-14
0.10	1.115170918075647700	1.115170918075798300	1.6159E-11	1.505462E-13