

FPGA Implementation of Novel Synchronization Methodology for a New Chaotic System

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Abstract

Presented in this paper, is an adaptive control scheme and a parameters update law based upon the Lyapunov stability theorem. It is intended for the synchronization of a new chaotic system with unknown parameters. Feed back control system for synchronization of chaotic system is proposed by using complete synchronization approach. In the proposed method, each system is constructed with MATLAB simulink blocks, and then Simulink designs are translated into System Generator design so that bitstream file used to program FPGA is obtained. Finally, the design is implemented into FPGA by downloading bitstream file into FPGA. As an application of FPGAs the synchronization of chaotic systems has been achieved. Additionally, we show the effectiveness of proposed method by the use of numerical solution.

Keywords: Adaptive Control, Chaos Synchronization, Complete Synchronization, FPGA Implementation, Lyapunov Stability Theorem, System Generator

1. Introduction

Study of the chaos synchronization has been the subject of numerous research efforts, since the work of Pecora and Carroll¹. It finds application in diverse areas including secure communication, biological systems, ecological systems, physical systems etc. The concept is to make the chaotic systems oscillate in a synchronized way. A variety of strategies were proposed for the synchronization of chaotic systems that include PC method¹, OGY method², Active control method³, Adaptive control method⁴, time delay feedback approach⁵, Back stepping method⁶, Sliding mode control⁷ etc. In many practical situations, parameters values are not well known. But most of the researchers involving chaotic systems require the values of these parameters. The uncertainty ultimately affects the synchronization largely. Hence, it is essential to design an

adaptive controller for the control and synchronization of chaotic systems with not well known parameters.

We will here forth, consider the chaos synchronization for chaotic systems with not well known parameters for master and slave systems based on Lyapunov stability theorem⁸. For the synchronization of chaotic systems with not well known parameters, a controller and parameters update law are designed.

2. Adaptive Complete Synchronization of Identical New Chaotic System

In this section, based on the adaptive control theory, the complete synchronization between two identical new chaotic system is achieved. The master system is given as follows,

$$\begin{aligned}\dot{r}_1 &= \alpha(w_1 - r_1) - \beta r_1^2 \\ \dot{w}_1 &= \gamma r_1 - \delta w_1 - r_1 b_1 \\ \dot{b}_1 &= r_1 w_1 - \sigma b_1\end{aligned}\quad (1)$$

The slave system is given as follows,

$$\begin{aligned}\dot{r}_2 &= \alpha(w_2 - r_2) - \beta r_2^2 + u_1 \\ \dot{w}_2 &= \gamma r_2 - \delta w_2 - r_2 b_2 + u_2 \\ \dot{b}_2 &= r_2 w_2 - \sigma b_2 + u_3\end{aligned}\quad (2)$$

Here r_1, w_1, b_1 , are the state variables of master system, r_2, w_2, b_2 , are the state variables of slave system, $\alpha, \beta, \gamma, \delta$ and σ are positive constant parameters u_1, u_2 and u_3 are control functions. The system is chaotic when $\alpha = 30, \beta = 0.48, \gamma = 80, \delta = 6$ and $\sigma = 5$.

Our goal is to determine the control function from adaptive control method. The complete synchronization error dynamics are defined as,

$$\dot{e}_1 = \dot{r}_2 - \dot{r}_1, \quad \dot{e}_2 = \dot{w}_2 - \dot{w}_1 \text{ and } \dot{e}_3 = \dot{b}_2 - \dot{b}_1$$

The Controller U is defined for the adaptive control functions such that the error dynamics drives to zero,

$$\begin{aligned}u_1(t) &= -\hat{\alpha}(e_2 - e_1) + \hat{\beta}(r_2^2 - r_1^2) - k_1 e_1 \\ u_2(t) &= -\hat{\gamma}e_1 + \hat{\delta}e_2 + r_2 b_2 - r_1 b_1 - k_2 e_2 \\ u_3(t) &= \hat{\sigma}e_3 - r_2 e_2 - k_3 e_3\end{aligned}\quad (3)$$

Where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta}$ and $\hat{\sigma}$ are estimate values of the not well known parameters $\alpha, \beta, \gamma, \delta$ and σ , respectively, and k_1, k_2 and k_3 are the positive constants. Now let us choose a parameters update law,

$$\begin{aligned}\dot{\hat{\alpha}} &= e_1(e_2 - e_1) \\ \dot{\hat{\beta}} &= -e_1(r_2^2 - r_1^2) \\ \dot{\hat{\gamma}} &= e_1 e_2 \\ \dot{\hat{\delta}} &= -e_2^2 \\ \dot{\hat{\sigma}} &= -e_3^2\end{aligned}\quad (4)$$

Consider a Lyapunov function candidate as,

$$V = \frac{1}{2} e^T e + \frac{1}{2(e_\alpha^2, e_\beta^2, e_\gamma^2, e_\delta^2, e_\sigma^2)}$$

$$\begin{aligned}\dot{V} &= e_\alpha[e_1(e_2 - e_1) - \dot{\hat{\alpha}}] + e_\beta[e_1(r_2^2 - r_1^2) - \dot{\hat{\beta}}] + e_\gamma[e_1 e_2 - \dot{\hat{\gamma}}] + \\ &\quad e_\delta[-e_2^2 - \dot{\hat{\delta}}] + e_\sigma[-e_3^2 - \dot{\hat{\sigma}}] - (k_1 e_1^2 + k_2 e_2^2 + k_3 e_3^2) \\ &= -(k_1 e_1^2 + k_2 e_2^2 + k_3 e_3^2)\end{aligned}$$

Thus, by Lyapunov stability theory, it is immediate that the synchronization error $e_i, i = 1, 2, 3$ and the parameter estimation error $e_\alpha, e_\beta, e_\gamma, e_\delta$ and e_σ decay to zero exponentially with time.

3. FPGA Implementation of the Novel Synchronization Methodology

The proposed novel Synchronisation methodology is implemented in Xilinx for obtaining results in FPGA. The first step is to construct the synchronized chaotic systems using MATLAB Simulink blocks. Then MATLAB Simulink blocks are translated in to Xilinx system generator blocks using Xilinx blockset library. After missing blocks in Xilinx blockset libraries are constructed, the design can be completed by arranging System Generator block in Xilinx blockset library. XST will be used to synthesize synchronized chaotic systems which were designed previously. Verilog Hardware description language (VHDL) is used to program FPGA. FPGA clock period can be arranged so that the desired outputs will be obtained. 100 ns is used as a clock period in all designs. The last thing which has to be arranged is Simulink system period section. It can be set as the same in Simulink fundamental sample time. After all requirements are done by pressing Generate button in System Generator block, ISE Project file will be obtained. From the ISE Project file, bitstream file is obtained. By loading bitstream file to FPGA, programming process of FPGA can be completed. We adopt the implementation with a fixed point and with a representation of the real data on 32 bits, 12 for the entire and 20 for the fraction.

3.1 Implementation of Master Chaotic System

The chaotic system (1) is implemented using Xilinx system generator blocks as in Figure 1. The initial conditions are chosen as $r_1(0) = 0, w_1(0) = 10$ and $b_1(0) = 4$. The phase portraits of the system (1) are given in Figure 2.

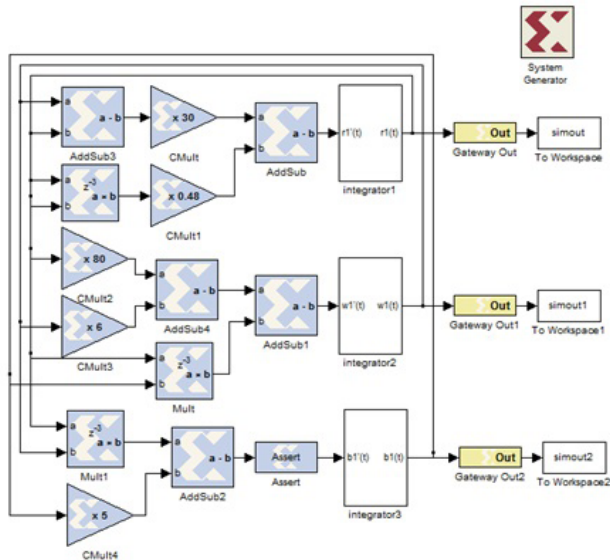


Figure 1. Implementation of Chaotic system (1) in System Generator.

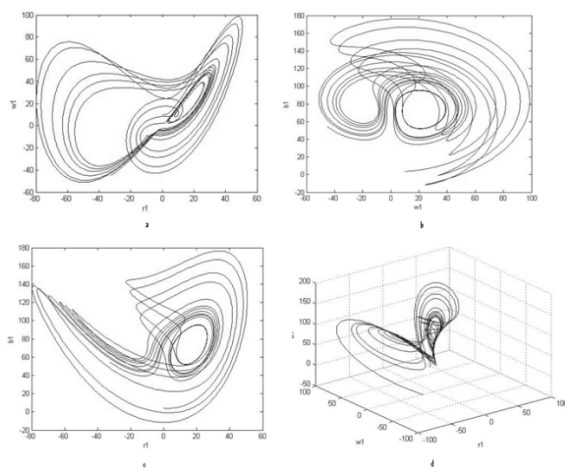


Figure 2. Phase Portrait of Chaotic system.

3.2 Implementation of Non Linear Adaptive Controller

The System generator implementation of Non linear adaptive controller (3) which is used to synchronize two chaotic systems is shown in Figure 3. The Figure 4 represents the time variation of adaptive controller u_1 , u_2 and u_3 . The Figure 4 also indicates that the control functions U reaches zero when the systems are synchronized.

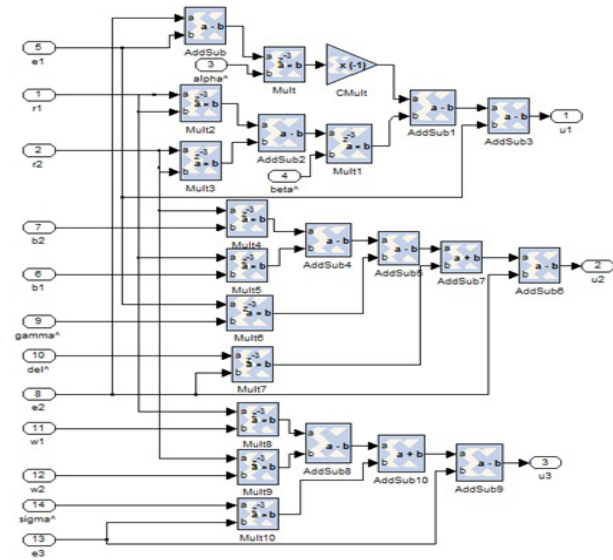


Figure 3. Implementation of Non Linear Adaptive Controller in System Generator.

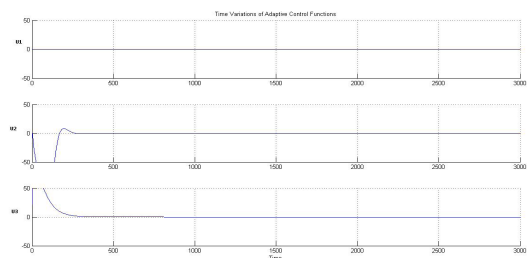


Figure 4. Time Variations of Adaptive Controller Function U .

3.3 Implementation of Parameter Update Law

The parameter update law (4) is implemented by system generator as in Figure 5. The initial values for not well known parameters α , β , γ , δ and σ are respectively taken as $\hat{\alpha} = 2$, $\hat{\beta} = -1$, $\hat{\gamma} = 3$, $\hat{\delta} = 4$ and $\hat{\sigma} = -5$.

3.4 Implementation of Synchronized Chaotic Systems

Figure 6 shows the system generator block diagram of synchronized chaotic systems. The block diagram has four subsystems. The master subsystem contains the typical chaotic generator (Figure 1), the slave subsystem contains the block diagram of slave system given in Equation 2. The parameter update law subsystem contains the Figure 5 and the control commands subsystem contains

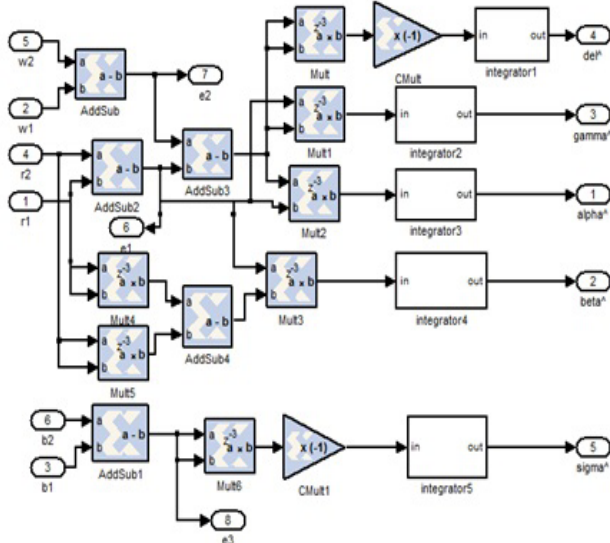


Figure 5. Implementation of Parameter Update Law.

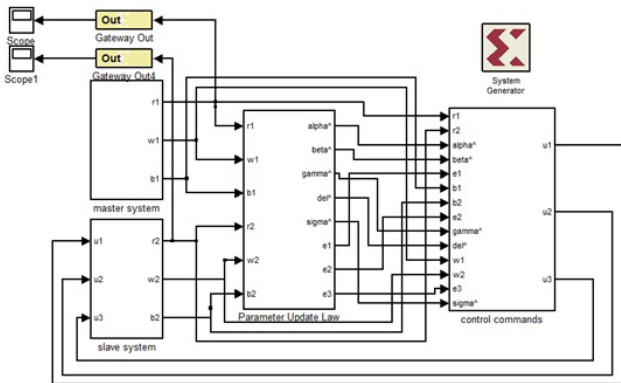


Figure 6. Implementation of Synchronized Chaotic System.

the Figure 3. The initial conditions for master and slave systems are chosen as $[r_1(0), w_1(0), b_1(0)] = [0, 10, 4]$ and $[r_2(0), w_2(0), b_2(0)] = [0, 10, -8]$ respectively. Figure 7, Figure 8 and Figure 9 represents the time evaluations of state variables r_1 , w_1 and b_1 of master system and the state variables of slave chaotic system r_2 , w_2 and b_2 respectively.

After implementing synchronized Hyperchaotic Lorentzsystem to FPGA, by using ISE program, how much source is consumed by synchronized Hyperchaotic Lorentzsystem can be seen in Table 1.

The implemented structure of the Hyperchaotic Lorentz systems in Xilinx ISE into Virtex-xc6vsx315t-3ff1156 is shown in the Figure10.

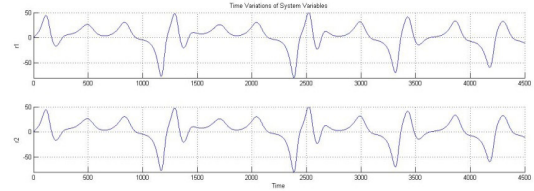


Figure 7. Master and Slave System Variables r_1 and r_2 .

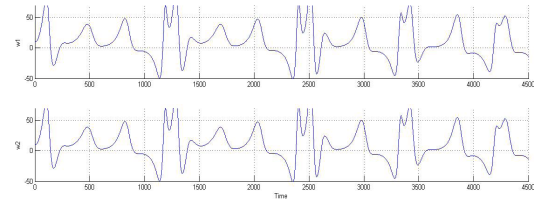


Figure 8. Master and Slave System Variables w_1 and w_2 .

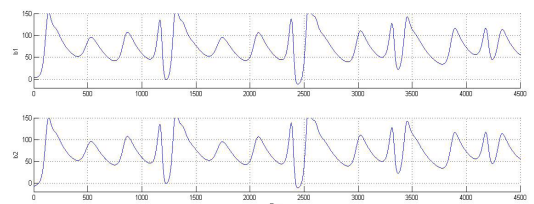


Figure 9. Master and Slave System Variables b_1 and b_2 .

Table 1. Utilisation of resources for virtex-xc6vsx315t-3ff1156

Number of Slice Registers	2,502	393,600	1%
Number of Slice LUTs	4,775	196,800	2%
Number of occupied Slices	1,466	49,200	2%
Number of bonded IOBs	193	600	32%
Number of RAMB36E1/FIFO36E1s	0	704	0%
Number of RAMB18E1/FIFO18E1s	0	1,408	0%
Number of BUFG/BUFGCTRLs	1	32	3%
Number of BUFOs	0	36	0%
Number of BUFRs	0	36	0%
Number of CAPTUREs	0	1	0%

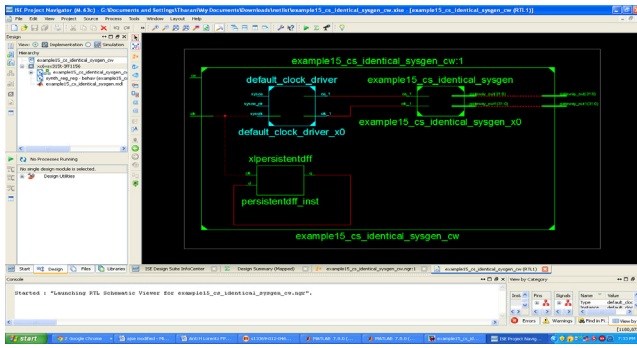


Figure 10. RTL Schematic of the Proposed Synchronisation Algorithm in Virtex xc6vsx315t-3ff1156.

4. Conclusion

In this paper, we proposed a complete synchronization methodology for chaotic system synchronization in FPGA via adaptive non linear control. The proposed method in this paper has been established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the adaptive control method is efficient for the complete synchronization of two identical chaotic systems. The chaotic system, controller, Parameter update law and the synchronized chaotic systems are implemented in FPGA using Xilinx system generator. The numerical simulations results are indicating that the proposed methodology is very effective and convenient to synchronize the chaotic systems using FPGA.

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