Indian Journal of Science and Technology, *Vol 8(10), 975–981, May 2015*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645
DOI: 10.17485/ijst/2015/v8i10/51936

A Model for Weibull Deteriorate Items with Price Dependent Demand Rate and Inflation

Rekha Rani Chaudhary[1] and Vikas Sharma[2*]

[1]Department of Mathematics, Government Engineering College Bharatpur, Rajasthan - 321001, Maharashtra, India; Rekhaparth2002@yahoo.co

[2]Banasthali Vidyapith, Rajasthan - 304022, Maharashtra, India; Vikasgaur01@gmail.com

Abstract

The present paper deal with an inventory model which is developed for most of the physical goods undergoes decay or deterioration over time. Commodities such as fruits, vegetables and foodstuffs suffer from depletion by direct spoilage while kept in store. The deterioration rate follows the Weibull distribution with two parameters. Demand rate is assumed as price dependent in linear form. It is an ordinary fact that unique price of items attracts more customers. The model is solved numerically by when shortages are partially backlogged with inflation. Analytical and numerical solution of the model is obtained to verify the optimal solution. Comprehensive sensitivity analysis has been given for viewing the result of variation in the parameter. The model is solved analytically by maximizing the total profit.

Keywords: Deteriorating Items, EOQ Model, Inventory, Price Dependent Demand Rate

1. Introduction

It has been seen that delivery of the goods is affected by the price dependent demand of the product. The attractive price of the items motivates the customer to buy more goods and that situation creates the greater demand of the goods. This condition motivates the retailer to increase their order quantity and the retailers earn the more profit to increase their revenue. The condition become more complex when the inventory is deteriorates in nature. Deterioration of goods is one of the important factors in any inventory and production system.

The several researchers have worked for inventory with deteriorating items in recent years because most of the physical goods undergo decay or deterioration over time. A model is developed for an exponentially decaying inventory[1]. Inventory models with a time dependent rate of deterioration were considered[2]. A deterministic lot-size inventory model for deteriorating with shortages and a declining market was introduced by Wee[3]. Deterministic models are suggested for perishable inventory with stock dependent rate and non-linear holding cost[4]. Some of the significant recent work has done in this field of Structural properties of an inventory system with deterioration and trended demand[5]. Optimum ordering policy is developed for decaying items with stock dependent demand under inflation in a supply chain[6]. A production-inventory problem of a product is introduced in inventory system with time varying demand as well as production and deterioration rates also[7]. An integrated production inventory model is defined with Expiration date and uncertain lead time in an inventory system[8]. Inventory model for ameliorating items for price dependent demand rate was proposed[9]. A production-inventory model for a deteriorating item is introduced with trended demand and shortages[10]. An ordering policy is formulated with stock dependent demand with partial backlogging and inflation[11]. Shortages were also considered in this model by them. An inventory Model for deteriorating items is proposed with Weibull deterioration with time dependent demand and Shortages[12]. Integrated vendor-buyer cooperative model is developed with multivariate demand and

**Author for correspondence*

progressive credit period[13]. Optimal Inventory Model with Weibull deterioration with Trapezoidal demand and shortages was introduced in this system[14]. The holding cost considered as a constant. Optimal ordering policy for deteriorating items is proposed with stock dependent demand under two-warehouse facility[15]. An inventory model for deteriorating items is introduced with shortages and time varying demand[16]. A Multi Item Inventory Model for Deteriorating Items with Expiration Date and Allowable Shortages[17] Developing an Inventory Mathematical Model with Deterioration Variables for Discounted Stochastic Goods[18]. An inventory model for decaying items, considering multi variate consumption rate with partial backlogging[19]. Multilevel inventory techniques for minimizing cost-a case study[20].

In most of the research paper demand rate is considered as time dependent in all situations. But in realistic demand is not always dependent on time. A discount price attracts more customers to buy the product in a super market.

In this model, we consider here an optimal inventory model for Weibull deteriorating items. Demand rate is price dependent in linear form.

This paper is formulated with Selling Price Dependent Demand with partially backlogging under Inflation.We solve the model to optimize the total profit. Model is illustrated with numerical examples and Comprehensive sensitivity analysis.

2. Assumptions and Notations

The proposed inventory model is developed under the following assumptions and notations:

2.1 Assumptions

The following assumptions are made for development of mathematical model:

- Demand rate is considered as selling price dependent of the items in linear form

  $D(s) = a - bs$

Where, $a > 0$, $0 \leq b \leq 1$, b is the selling price dependent demand rate parameter.

- The ordering cost A_0 is constant.
- The inflation is also consider, the inflation rate is assume r.

  Which is define as follows:

$$f(t) = e^{-rt} \quad r > 0$$

- The cycle length is assumed $0 < t < T$.

2.2 Notations

The following notations are made for development of mathematical model:

- **I(t)** be the inventory level at time t ($0 \leq t < T$).
- **T** is the cycle length.
- **q** is The ordering quantity is.
- $\mathbf{A_0}$ is the ordering cost.
- **s** is the selling price per unit item .
- **h** is the inventory holding cost per unit item per unit time.
- $\mathbf{C_1}$ is the shortage cost per unit item per unit time.
- $\mathbf{C_2}$ is the deterioration cost per unit item per unit time.
- $\mathbf{C_3}$ is the lost sale cost per unit item per unit time.
- $D(t) = a - bs$ ($a > 0$, $b > 0$) is the Demand rate.
- The deterioration of time as follows by Weibull parameter (two) distribution $\theta(t) = \alpha\beta t^{(\beta-1)}$ where, $0<\alpha<1$ is the scale parameter and $\beta > 0$ is the shape parameter.
- **r** is the inflation rate .
- γ is the partially backlogging rate , *where* $0 < \gamma < 1$.
- **Q** is the optimal inventory level.
- **S(T, s)** is expected total profit.

3. Formulation of the Model with Selling Price Dependent Demand with Partially Backlogged Shortages under Inflation

The inventory is depleting during time t_1 Due to the demand and deterioration of the items. The inventory level becomes zero at the time t_1 and shortages are occurs in the period (t_1, T) which are partially backlogged. The length of the cycle is T. Let I(t) be the inventory level at time t ($0 \leq t < T$).

Differential equations of the inventory system

$$I'(t) + \alpha\beta t^{(\beta-1)} I(t) == -(a - b.s) \quad 0 \leq t \leq t_1 \tag{3.1}$$

$$I'(t) = -(a - b.s) \quad t_1 \leq t \leq T \tag{3.2}$$

With $I(t_1) = 0$ at $t = t_1$

From equation (3.1) we get:

$$I(t)=[1-\alpha t^{\beta}]\Bigg[a(t_1-t)-bs(t_1-t)+\frac{a\alpha}{(\beta+1)}\left(t_1^{(\beta+1)}-t^{(\beta+1)}\right)-\frac{bs\alpha}{(\beta+1)}\left(t_1^{(\beta+1)}-t^{(\beta+1)}\right)\Bigg]\; 0\le t\le t_1 \tag{3.3}$$

From equation (3.2) we get

$$I(t)=\left[a(t_1-t)-bs(t_1-t)\right]\; t_1\le t\le T \tag{3.4}$$

3.1 Expected Holding Cost under the Inflation Rate r

The holding cost during the time period 0 to t_1

$$H=\int_0^{t_1} e^{-rt}.I(t)dt$$

The total holding cost during the time period 0 to t_1

$$H=h.\int_0^{t_1} e^{-rt}.I(t)dt$$

$$H=h\int_0^{t_1} e^{-rt}.\left[1-\alpha t^{\beta}\right]\Bigg[a(t_1-t)-bs(t_1-t)+\frac{a\alpha}{(\beta+1)}\left(t_1^{(\beta+1)}-t^{(\beta+1)}\right)-\frac{bs\alpha}{(\beta+1)}\left(t_1^{\beta+1}-t^{(\beta+1)}\right)\Bigg]dt$$

Now total holding cost will be:

$$H=h(a-bs)\Bigg[\frac{1}{2}t_1^2+\frac{r}{6}t_1^3-\frac{\alpha}{(\beta+1)(\beta+2)}t_1^{(\beta+2)}+\frac{\alpha r}{(\beta+2)(\beta+3)}t_1^{(\beta+3)}+\frac{\alpha}{(\beta+2)}t_1^{(\beta+2)}-\frac{\alpha r}{(\beta+3)}t_1^{(\beta+3)}-\frac{\alpha^2}{2(\beta+1)^2}t_1^{(2\beta+2)}+\frac{\alpha^2 r}{(\beta+2)(2\beta+3)}t_1^{(2\beta+3)} \tag{3.5}$$

3.2 Expected Deterioration Cost under the Inflation Rate r

The total deterioration cost for the period of 0 to t_1 is given by,

$$D_T=C_2\int_0^{t_2} e^{-rt}.\theta(t).I(t)dt$$

$$D_T=C_2\int_0^{t_2} e^{-rt}.\alpha\beta t^{(\beta-1)}.\left[1-\alpha t^{\beta}\right]\Bigg[a(t_1-t)-bs(t_1-t)+\frac{a\alpha}{(\beta+1)}\left(t_1^{(\beta+1)}-t^{(\beta+1)}\right)-\frac{bs\alpha}{((\beta+1))\left(t_1^{(\beta+1)}-t^{(\beta+1)}\right)}\Bigg]dt$$

Now total deterioration cost will be,

$$D_T=C_2\alpha\beta(a-bs)\Bigg[\frac{1}{\beta(\beta+1)}t_1^{(\beta+1)}-\frac{r}{(\beta+1)(\beta+2)}t_1^{(\beta+2)}-\frac{\alpha}{2\beta(2\beta+1)}t_1^{(2\beta+1)}+\frac{\alpha r}{(2\beta+1)(2\beta+2)}t_1^{(2\beta+2)}+\frac{\alpha}{\beta(2\beta+1)}t_1^{(2\beta+1)}+\frac{\alpha r}{2(\beta+1)^2}t_1^{(2\beta+2)}-\frac{\alpha^2}{2\beta(3\beta+1)}t_1^{(3\beta+1)}+\frac{\alpha^2 r}{(2\beta+1)(3\beta+1)}t_1^{(3\beta+2)}\Bigg] \tag{3.6}$$

3.3 Expected Shortage Cost Under the Inflation Rate r

The total shortage cost for the period t_1 to T is given by,

$$sh=-C_1\int_{t_1}^{T} e^{-rt}.I(t)dt$$

$$sh=-C_1\int_{t_1}^{T} e^{-rt}.\left[a(t_1-t)-bs(t_1-t)\right]dt$$

$$sh=C_1(a-bs)\left[T^2+t_1^2-2Tt_1+\frac{r}{2}\left(T^2t_1-T^3-t_1^3+t_1^2T\right)\right] \tag{3.7}$$

3.4 Expected Lost Sale Cost under the Inflation Rate r

$$\text{Lost sale cost}=-c_3\int_{t_1}^{T} e^{-rt}.(1-\gamma)(a-bs)dt$$

$$Lost\ sale\ cost = c_3 b(1-\gamma)(a-bs)\left[(t_1 - T) - \frac{r}{2}\left(t_1^2 - T^2\right)\right] \tag{3.8}$$

3.5 Expected Total Profit

From the above equations the total profit per unit time can describe:

$$\begin{aligned} S(T,s) &= s.(a-bs) - \frac{1}{T}\big[Ordering\ cost + Expected\ holding\ cost \\ &+ Expected\ deterioration\ cost + Expected\ shortage\ cost \\ &+ Lost\ sale\ cost\big] \end{aligned}$$

$$\begin{aligned} S(T,s) &= s(a-bs) - \frac{1}{T}\Big\{A_0 + h(a-bs)\Big[\frac{1}{2}t_1^2 + \frac{r}{6}t_1^3 \\ &- \frac{\alpha}{(\beta+1)(\beta+2)}t_1^{(\beta+2)} + \frac{\alpha r}{(\beta+2)(\beta+3)}t_1^{(\beta+3)} + \frac{\alpha}{(\beta+2)}t_1^{(\beta+2)} \\ &- \frac{\alpha r}{(\beta+3)}t_1^{(\beta+3)} - \frac{\alpha^2}{2(\beta+1)^2}t_1^{2(\beta+2)} + \frac{\alpha^2 r}{(\beta+2)(2\beta+3)}t_1^{(2\beta+3)}\Big] \\ &+ C_2\alpha\beta(a-bs)\Big[\frac{1}{\beta(\beta+1)}t_1^{(\beta+1)} - \frac{r}{(\beta+1)(\beta+2)}t_1^{(\beta+2)} \\ &- \frac{\alpha}{2\beta(2\beta+1)}t_1^{(2\beta+1)} + \frac{\alpha r}{(2\beta+1)(2\beta+2)}t_1^{(2\beta+1)} + \frac{\alpha}{\beta(2\beta+1)}t_1^{(2\beta+1)} \\ &- \frac{\alpha r}{2(\beta+1)^2}t_1^{(2\beta+2)} - \frac{\alpha^2}{2\beta(3\beta+1)}t_1^{(3\beta+1)} + \frac{\alpha^2 r}{(2\beta+1)(3\beta+2)}t_1^{(3\beta+2)}\Big] \\ &+ C_1(a-bs)\Big[T^2 + t_1^2 - 2Tt_1 + \frac{r}{2}\left(T^2 t_1 - T^3 - t_1^3 + t_1^2 T\right)\Big] \\ &+ c_3 b(1-\gamma)(a-bs)\Big[(t_1 - T) - \frac{r}{2}\left(t_1^2 - T^2\right)\Big]\Big\} \end{aligned} \tag{3.9}$$

3.6 Mathematical Formulation of the Model

Our key purpose to maximize the profit function **S(T, t$_1$)** the necessary condition for maximize the profit are $\frac{\partial S(T,t_1)}{\partial T} = 0\ and\ \frac{\partial S(T,t_1)}{\partial s} = 0$

$$\frac{\partial S(T,s)}{\partial T} = 0$$

$$\begin{aligned} &+\frac{1}{T^2}\Big\{A_0 + h(a-bs)\Big[\frac{1}{2}t_1^2 + \frac{r}{6}t_1^3 - \frac{\alpha}{(\beta+1)(\beta+2)}t_1^{(\beta+2)} \\ &+ \frac{\alpha r}{(\beta+2)(\beta+3)}t_1^{(\beta+3)} + \frac{\alpha}{(\beta+2)}t_1^{(\beta+2)} - \frac{\alpha r}{(\beta+3)}t_1^{(\beta+3)} \\ &- \frac{\alpha^2}{2(\beta+1)^2}t_1^{(2\beta+2)} + \frac{\alpha^2 r}{(\beta+2)(2\beta+3)}t_1^{(2\beta+3)}\Big] \\ &+ C_2\alpha\beta(a-bs)\Big[\frac{1}{\beta(\beta+1)}t_1^{(\beta+1)} - \frac{r}{(\beta+1)(\beta+2)}t_1^{(\beta+2)} \\ &- \frac{\alpha}{2\beta(2\beta+1)}t_1^{(2\beta+1)} + \frac{\alpha r}{(2\beta+1)(2\beta+2)}t_1^{(2\beta+1)} \\ &+ \frac{\alpha}{\beta(2\beta+1)}t_1^{(2\beta+1)} - \frac{\alpha r}{2(\beta+1)^2}t_1^{(2\beta+2)} \\ &- \frac{\alpha^2}{2\beta(3\beta+1)}t_1^{(3\beta+1)} + \frac{\alpha^2 r}{(2\beta+1)(3\beta+2)}t_1^{(3\beta+2)}\Big]\Big\} \\ &C_1(a-bs)\Big[1 - \frac{t_1^2}{T^2} + \frac{r}{2}\Big(t_1 - 2T + \frac{t_1^3}{T^2}\Big)\Big] \\ &+ c_3 b(1-\gamma)(a-bs)\Big[\frac{t_1}{T^2} - \frac{r}{2}\Big(\frac{t_1^2}{T^2} + 1\Big)\Big] = 0 \end{aligned} \tag{3.10}$$

$$\text{And } \frac{\partial S(T,s)}{\partial s} = 0 \tag{3.11}$$

The optimal value of T* and s* can be calculated simultaneously by equation (3.10) and equation (3.11) by using the **Mathematica 8.0**. The optimal values of profit function **S(T, s)** under inflation also determined by equation (3.9). Sufficient conditions for maximizing profit function S (T, s) are satisfied by the optimal value of T* and s*.

$$\frac{\partial^2 S(T,s)}{\partial T^2} < 0,\ \frac{\partial^2 S(T,s)}{\partial s^2} < 0 \tag{3.12}$$

And $\frac{\partial^2 S(T,t_1)}{\partial T^2}.\frac{\partial^2 S(T,t_1)}{\partial s^2} - \frac{\partial^2 S(T,t_1)}{\partial T \partial s} > 0$ And at T = T* optimal value s = s* (3.13)

3.7 Numerical Illustration

The model has been explored numerically as well. We have considered the following data for the study which based on previous study.

Example: Let us consider, $A = 750$, $a = 200$, $b = 0.25$, $\alpha = 0.1$, $\beta = 1.3$, $h = 1.2$, $C_1 = 1.1$, $C_2 = 0.02$, $t_1 = 1.25$, $r = 0.5$, $\gamma = 0.2$

We solve the problem numerically with the help of computer software **Mathematica-8.0**, and calculate the optimal value of S(T,s), T∗ and s∗ simultaneously by equation (3.9), equation (3.10) and equation (3.11).

S (T, s) = 1582.63, T∗ = 3.24169, s∗ = 9.8325.

3.8 Sensitivity Analysis and Observations

We have study the effects of changes of the parameters on the optimal values of S(T,s), T∗ and s∗ derived by proposed method. The sensitivity analysis is performed in view of the numerical example given above. We have executed sensitivity analysis by changing the parameters *a*, *α*, *β* and b as +20%, +50%, −20% and −50%. All remaining parameters have original values with respect to these changes. The corresponding changes in S (T, s), T∗ and s∗ are showed in below (Table 1).

Table 1. Sensitivity analysis of optimal solution {S(T, s)} w.r.t. various parameters

parameters	% change	T∗	s∗	S (T, s)
a	−50	4.63793	10.60215	79.1315
	−20	3.80958	10.05240	791.315
	+20	2.59218	9.66750	2579.6869
	+50	1.80571	9.79768	4510.4955
α	−50	3.28645	10.84990	1609.5347
	−20	3.25390	10.43818	1593.8666
	+20	3.15173	9.22810	1571.3933
	+50	3.17248	8.72388	1555.7257
β	−50	3.39729	11.92046	1584.6804
	−20	3.30004	10.45436	1583.3134
	+20	3.20408	9.65643	1582.7047
	+50	3.10878	9.56719	1582.7595
b	−50	3.57634	10.47387	332.3532
	−20	3.41023	10.97911	923.1480
	+20	3.14012	9.69506	2019.8802
	+50	2.97876	9.80501	2405.5976

We study above (Table 1.) brings out the following.

We observed that as parameter *a* and *b* increase, optimal value of T∗ and s∗ decrease while the average total profit S (T, s) of an inventory system increases, whereas parameter *a* and *b* decrease, optimal value of T∗ and s∗ increase while the average total profit S (T, s) of an inventory system decreases. It is interesting to observe that deterioration parameter *α* increase, optimal value of T∗ and s∗ decrease while the average total profit S (T, s) of an inventory system decreases. If deterioration parameter *α* decrease, optimal value of T∗ and s∗ increase while the average total profit S (T, s) of an inventory system increases. Second deterioration parameter *β* increase, optimal value of T∗ and s∗ decrease while the average total profit S (T, s) of an inventory system slightly increases. If deterioration parameter *β* decrease, optimal value of T∗ and s∗ increase while the average total profit S (T, s) of an inventory system increases.

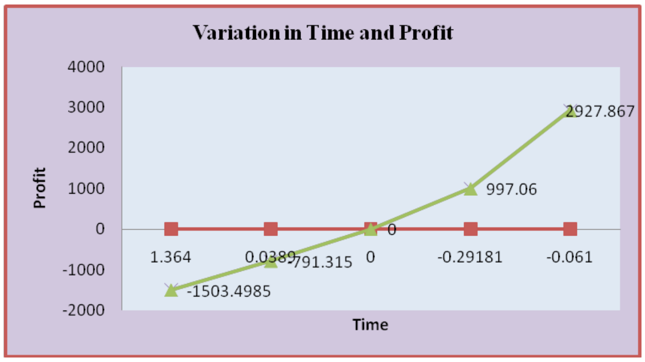

Figure 1. Graphical representation of sensitivity of the time and profit w.r.t. α.

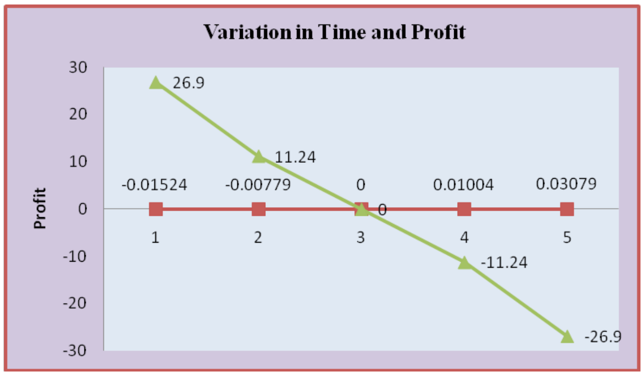

Figure 2. Graphical representation of sensitivity of the time and profit w.r.t. α.

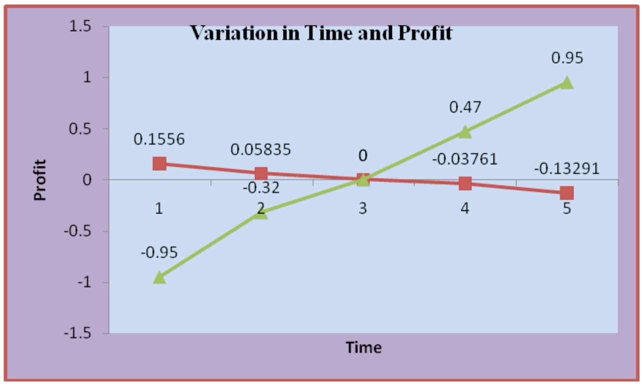

Figure 3. Graphical representation of sensitivity of the time and profit w.r.t. β.

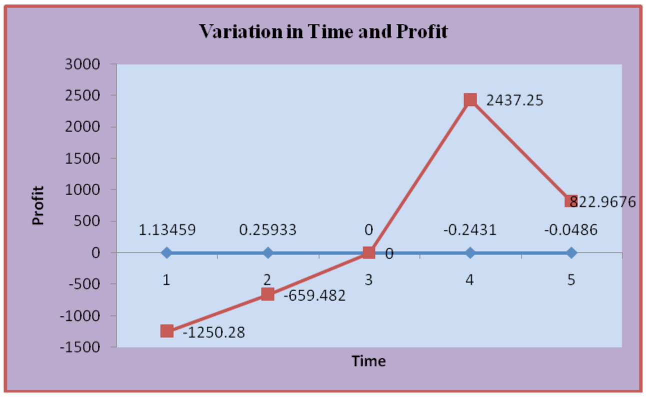

Figure 4. Graphical representation of sensitivity of the Time and profit w.r.t. β.

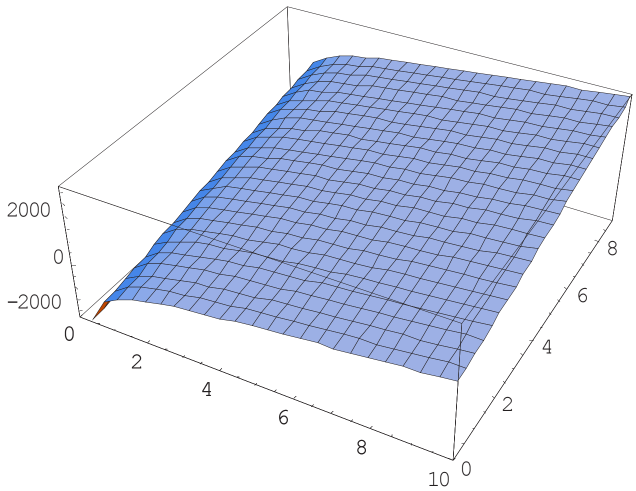

Figure 5. Graphical representation of concavity of the Profit function.

3.9 Graphical Representation of Concavity of the Profit Function

We get a three dimensional concave graph of total profit given by the following (Figure 5).

4. Conclusion

In this paper, we have developed an inventory lot size model for deteriorating items with Weibull deterioration rate which follows the distribution with two parameters. The demand rate is assumed of price dependent. The shortages are allowed and shortages are partially backlogged. The inflation is also considered in this model. The deterioration cost, inventory holding cost and, shortage cost are considered with inflation in this model. Manager of the industry always take care of selling price parameters which affect the profit quickly. The numerical examples are given to illustrate the development of model. Comprehensive sensitivity analysis (Table 1.) has been carried out for showing the effect of variation in the parameter. The model is solved analytically by maximizing the total profit. In the numerical examples we found the maximum value of profit.

5. References

1. Ghare PM, Schrader GP. A model for exponentially decaying inventories. Journal of Industrial Engineering. 1963; 14:238–43.
2. Covert RP, Philip GC. An EOQ model for items with Weibull distribution deterioration. AIIE Transaction. 1973; 5(4):323–6.
3. Wee HM. A deterministic lot-size inventory model for deteriorating with shortages and a declining market. Computers and Operations. 1995; 22(3):345–56.
4. Giri BC, Chaudhari KS. Deterministic models of perishable inventory with stock dependent rate and non-linear holding cost. Eur J Oper Res. 1998; 105(3):467–77.
5. Jalan AK, Chaudhari KS. Structural properties of an inventory system with deterioration and trended demand. Int J Syst Sci. 1999; 30:627–33.
6. Singh SR, Singh C, Singh TJ. Optimum ordering policy for decaying items with stock dependent demand under inflation in a supply chain. International Review of Pure and applied Mathematics. 2007; 3(2):189–97.
7. Goyal SK, Giri BC. The production-inventory problem of a product with time varying demand, production and deterioration rates. Eur J Oper Res. 2003; 147(3):549–57.
8. Singh SR, Chaudhary U. An integrated production inventory model with expiration date and uncertain lead time in a supply chain. International Transactions in Mathematical Sciences and Computer Applications. 2008; 1(1):187–202.
9. Mondal B, Bhunia AK, Maiti M. An inventory system of ameliorating items for price dependent demand rate. Comput Ind Eng. 2003; 45(3):443–56.
10. Sana S, Goyal SK, Chaudhuri KS. A production-inventory model for a deteriorating item with trended demand and shortages. Eur J Oper Res. 2004; 157(2):357–71.
11. Singh SR, Pandey RK, Kumar S. An ordering policy for perishable items having stock dependent demand with partial backlogging and inflation. International Journal of Mathematics, Computer Science and Technology. 2008; 1(1–2):239–44.

12. Chaudhary RR, Sharma V. An inventory model for deteriorating items with Weibull deterioration with time dependent demand and shortages. Res J Manag Sci. 2013; 2(3):1–4.
13. Singh SR, Diksha. Integrated vendor-buyer cooperative model with multivariate demand and progressive credit period. The ICFAI University Journal of Operations Management. 2009; 8(2):36–50.
14. Chaudhary RR, Sharma V. Optimal inventory model with Weibull deterioration with trapezoidal demand and shortages. Int J Eng Res Tech. 2013; 2(3):1–10.
15. Singh SR, Singh C. Optimal ordering policy for deteriorating items with stock dependent demand under two-warehouse facility. Opsearch. 2013; 50(2):182–96.
16. Sicilia J, Acosta JF, Pablo DA. An inventory model for deteriorating items with shortages and time varying demand. Int J Prod Econ. 2014.
17. Singh SR, Tayal S, Sharma R. A multi item inventory model for deteriorating items with expiration date and allowable shortages. Indian Journal of Science and Technology. 2014; 7(4):463–71.
18. Shahraki MR, Shahraki A, Javdan N. Developing an inventory mathematical model with deterioration variables for discounted stochastic goods. Indian Journal of Science and Technology. 2014; 7(11):1750–5.
19. Singh SR, Sharma S. An inventory model for decaying items, considering multivariate consumption rate with partial backlogging. Indian Journal of Science and Technology. 2013; 6(7):7870–80.
20. Bhagoria M, Sadiwala CM, Khare VK. Multilevel inventory techniques for minimizing cost-a case study. Indian Journal of Science and Technology. 2010; 3(6):693–5.