

Enhancing Students' Level of Geometric Thinking Through Van Hiele's Phase-based Learning

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Abstract

The purpose of this study was to test the effectiveness of Van Hiele's phase-based learning on students' levels of geometric thinking. Quasi-experimental design was used in this study. The six-week study was conducted in a secondary school involving 94 students and two teachers. The students were divided into two groups, with 47 in a control group and the other 47 in a treatment group. Van Hiele's Geometry Test (VHGT) was given to both groups before and after the treatment. Ten students were randomly selected to further determine their initial and final levels of geometric thinking. Wilcoxon-t tests were conducted to test the developed hypotheses. The results showed that there was no significant difference between the initial levels of geometric thinking in both groups. However, the analysis showed significant difference between the final levels of geometric thinking in both groups. Furthermore, qualitative analysis revealed that, in initial levels of geometric thinking, the majority of students in both groups obtained the first Van Hiele levels with complete acquisition, a low acquisition of level two and no acquisition of level three. In the post interview, most of the students in the control group showed an increment of geometric thinking from level one to level two, but no one in this group achieved level three. In contrast, all the students in the treatment group showed a complete acquisition of Van Hiele level one and almost all of them indicated a complete acquisition of level two. As for level three, only one student did not achieve this level, whereas the rest showed a complete and high level of acquisition. This demonstrates that Van Hiele's phase-based learning can be applied in classrooms in order to help students achieve better level of geometric thinking.

Keywords: Van Hiele's Phase-based Learning, Students' Levels of Geometric Thinking, Van Hiele Model, Learning Geometry.

1. Introduction

In the Malaysian education system, geometry-related topics are emphasised in the syllabus at secondary school level. 42% of the 60 topics in the Integrated Curriculum for Secondary School (KSBM) of Mathematics from Form One to Form Five consist of geometry topics [9]. In fact, students are exposed formally to the geometry concepts for two- and three-dimensional shapes as early as Year One in these topics [11]. Current teaching and learning

practice in the classroom does not reflect the importance of geometry in the lives of students, and the emphasis that is placed on it in the mathematics curriculum. Teacher training is still bound to the traditional approach that is teacher-centred [21, 22, 23, 25]. According to Wan Mohd Rani [37], in terms of teacher training and attitude, more often teachers who teach mathematics use the blackboard to explain theorems, definitions, and concepts, and to show the solutions for the related problems. Students are commonly shown methods and

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algorithms, which they then memorised without actually understanding the concepts [17]. Mathematics teachers' practice in Malaysia can also be seen from the research done by TIMSS conducted in 1999, 2003, and 2007 [21, 22, 23].

From the published reports, we can see similar trends. In the study conducted in 1999, majority of the students stated that a lot of time is spent in mathematics lessons listening to the concepts explained by the teacher [21]. In the study conducted in 2003, the highest percentage of time taken by the students in mathematics lesson in a week was listening to what the teacher was saying and then solving the mathematical problems with guidance from the teachers. In the study conducted in 2007, the highest percentage of time taken by the students in mathematics class was for listening to the lecture delivered by the teacher, which scored 22% and was followed by solving mathematical problems with the guide from the teacher, which scored 18%, and finally there was the discussion of mathematical problems with guidance from the teacher and the solving mathematical problems without the guidance, both scored the same percentage, 13% [23]. In TIMSS 2007 report [23], the percentage of Form Two students in Malaysia stating that they memorised formulae and procedures as an activity that spent half or more of the time in mathematics class was as high as 69%. This was followed by explaining the answers (61%), relating the subjects learnt with daily life (55%), solving the problems on their own (48%), and identifying procedures to solve complex problems (36%). Furthermore, the percentages of students memorising the formulae and procedures, applying facts, concepts, and procedures to solve routine questions, and explaining answers, as reported by the teacher, were high compared to other activities, which are 58%, 65%, and 75%, respectively. These practices do not encourage the students to think, which is one of the aspects that need to be emphasised especially in mathematics. According to Curriculum Development Centre (CDC) [12], in mathematics, students are taught to think, to have wide knowledge, to become highly ethical and smart, and to be able to use information and communication technology effectively. Mathematics is also a discipline that trains the students to think logically and systematically in solving problems and making decisions. According to CDC [12], the things that need to be given special focus in the process of teaching and learning is logical, systematic and creative mental development and valid reasoning to produce a person who can think logically and rationally.

2. Van Hiele's Geometric Thinking Level and Phase-based Learning

In the field of geometry, the best and most well-defined model for student levels of thinking is based Van Hiele's model [2, 29]. The levels are visualisation, analysis, informal deduction, formal deduction, and rigor. The first Van Hiele's level of thinking is known as visualisation level. At this level, students are able to recognise geometric shapes. The second level in the model is known as analysis level where students are able to identify the properties of certain shapes. The third level in the model is informal deduction where students are able to comprehend the relation between shapes and create the relationships. The fourth level in the model is formal deduction. At this level, students can appreciate the meaning and importance of deduction and the role of postulates, theorems, and proofs. Finally, the fifth level in Van Hiele's model is rigor. At this level, students come to understand how to work in an axiomatic system. They are able to make more abstract deductions. However, lower secondary school students can usually only achieve up to the third level in Van Hiele's model, which is informal deduction [16, 28, 34, 35].

Van Hiele's model proposes learning phases that are able to help in assisting students to move from one of Van Hiele's levels of geometric thinking to a higher level [8, 26]. These learning phases can assist students in learning geometry and, with assistance from teachers, they will be able to discuss certain concepts and develop a more technical use of language [30]. The approach used in these five phases provides a structured lesson. Based on Crowley [8], in the information phase, the interaction between teacher and students through discussion is emphasised. In the guided orientation phase, students make discoveries using guided activity. In the explicitation phase, students explain and express their views about the observed structure. In the free orientation phase, students solve more complex tasks. In the integration phase, students summaries the lesson learnt for the purpose of establishing a new overall view. According to Chew [4] and Choi-Koh [5], students must go through all the five phases in order to achieve each of Van Hiele's level of geometric thinking. In other words, students must go through the information, guided orientation, explicitation, free orientation, and integration phases to advance from the first level to the second level, and then they have to go through the same phases to advance to the next stages. In this study, as shown in Figure 1, students have had to go through the phases twice to advance from

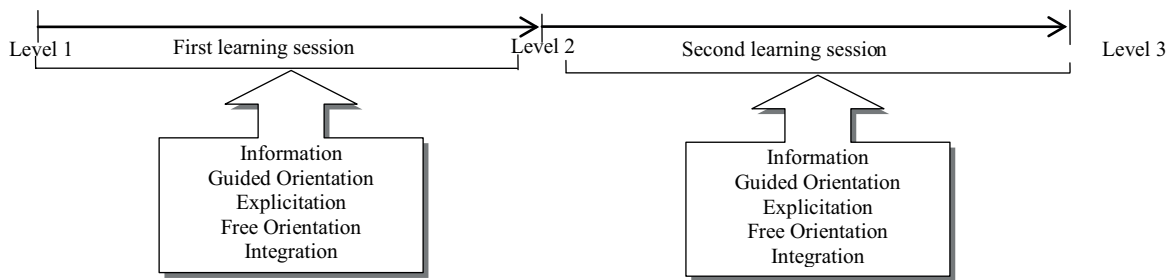


Figure 1. Phases of learning geometry.

first level to the second level and from the second level to the third level. This was due to the researcher's taking into account previous research which found that lower secondary school students usually can only reach up to Van Hiele's third level of geometric thinking, which is informal deduction [34, 35].

There are a few studies on the implementation of the phases of geometry learning which were done locally and globally. Tay [32] studied the effectiveness of the implementation of phases of geometry learning using manipulative materials to give students opportunities to explore and investigate the properties of geometric shapes. Shi-Pui and Ka-Luen [31] also implemented phases of geometry learning using manipulative materials in the solid geometry topic. Liu [18] studied the effectiveness of Van Hiele's phases of learning geometry in the Circle topic. He used worksheet to implement the phases. These studies [18, 32, 33] found that students in the treatment group who were exposed to Van Hiele's learning phases achieved a better Van Hiele's level of geometric thinking than the students in the control group who were exposed to the same learning topic but used traditional approaches. However, according to Tay [33], dynamic geometry software can be used to replace manipulative materials to give students the opportunity to explore the concepts of geometry. Selecting the appropriate and suitable technology would help students develop the ability to understand concepts of mathematics more thoroughly and greater pace [10]. One of them is dynamic geometry software which gives opportunities to students to explore geometry shapes intuitively and inductively [31]. Choi-Koh [5] developed activities based on Van Hiele's phases of learning geometry using Geometer's Sketchpad (GSP) software. The activities were conducted by students with the assistance of GSP software and covered the topic of triangles. Serow [31] also implemented a project that used the approach of these phases of learning by

including the elements of technology to assist the process of teaching and learning geometry in mathematics lessons. Topics included in this study were space and geometry, which included the subtopics of classification, construction and identification of the properties of triangle and quadrilateral and proof of properties of the quadrilaterals. Chew [4] conducted a research about the geometry of solids learning among Form One students in the learning environment based on Van Hiele's phases of geometric thinking using GSP software. The objectives of his study were to determine the initial Van Hiele's levels of Van Hiele's model on cubes and cuboids, and how students' levels of Van Hiele's model changed after being taught through the phases by using GSP. He found that the students' initial Van Hiele's levels of geometric thinking varied between level 1 and level 2. After the teaching based on phases using GSP, students' Van Hiele's levels of geometric thinking either increased or remained at the same level.

3. Example of Developed Activities Based on the Van Hiele's Phase-based Learning

Therefore, the researchers have developed activities for form two students based on the phases for the topic of Transformations. One of the subtopic is *Quadrilaterals: Their Properties and Relationship*. The following is an example of developed activities based on the Van Hiele's phase-based learning. As shown in Figure 1, students must go through the information, guided orientation, explication, free orientation and integration phases in the first learning session to advance from the first level to the second level of geometric thinking, and they have had to go through the same phases in the second learning session to advance to the next level. In this study, activities were

prepared to assist students to enhance to the third level of geometric thinking. This is due to the fact that much previous research revealed that lower secondary school students usually can only reach up to the third level of geometric thinking which is informal deduction [34, 35].

3.1 First Learning Session

In this session, learning activities are provided to help students advance from the first level of the Van Hiele Model; visualisation, to the second level; analysis. Students will go through all phases; visualization, guided orientation, explication, free orientation and integration to move from the first level to the second level. The objective of the activities is to help students identify quadrilaterals and to understand their properties. For example, students will come to know that a parallelogram has equal and parallel opposite sides, equal opposite angles and its diagonals bisect each other. In Phase 1 which is information, students will become acquainted with the activity. Teachers will present a new idea and allow students to begin working on the concept. In the example given by Noraini [27], shapes such as rhombus are introduced in this phase. Students are then introduced to other geometrical shapes and asked if the shapes are rhombus. In the study by Husnaeni [16], teachers gave a few figures of various shapes and asked the students to identify triangles and other shapes. A similar study by Choi-Koh [5] stated that in information phase, students were able to recognize and draw the shapes. They could identify the type of triangle, be it equilateral triangle, isosceles triangle, or right triangle. In the study by Liu [18], in the topic of Circles, students used their own description to name the sides in a circle in the information phase. They most probably named the sides based on their external properties. In this research, the available activities will help students to develop and recognise the variety of the quadrilaterals. For example, students can recognise that (a) rectangle, (b) square, (c) parallelogram; (d) rhombus and (e) kite (see Figure 2). By using the GSP, the students will then be able to construct quadrilaterals and then identify the properties they possess.

In Phase 2 which is guided orientation, students are given activities that allow them to become familiar with the many properties of the new geometric concept. They will carefully explore the objects used in the instruction. In this phase, students explore the properties of rhombus by folding a rhombus at its axial symmetry and by observing the diagonals and sides [27]. Meanwhile, in this

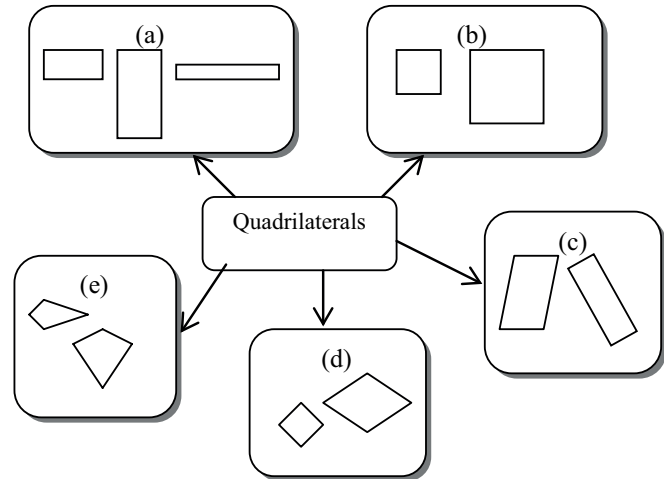


Figure 2. Types of quadrilaterals.

phase for triangles, Husnaeni [16] stated that students in groups were asked to observe figures of triangles and non-triangles. They were then asked to classify the figures into triangles and non-triangles. After that, they were asked to cut figures of triangles and draw the figures again in various sizes. The purpose of this activity is to help students explore the properties of the various types of triangles. In the study by Choi-Koh [5], students used the GSP software to explore the properties of equilateral triangle, isosceles triangle, and right triangle. In the study by Liu [18], in the topic of Circles, students were asked to measure the angles and state the relationship between the two angles. In this research, the activities will give students an opportunity to explore the properties possessed by any quadrilaterals by using the GSP. The processes of constructing quadrilaterals and exploring their properties can be done easily and effectively because the dragging capability of the GSP allows students to manipulate and reshape the geometrical objects with the use of the mouse. Without the use of any dynamic geometry software, students may find difficulties in constructing the shapes and getting the right values for their widths, lengths and angles. This is due to the weaknesses in construction and exploration when using paper, pencil and compass. For example, as shown in the Figure 3, when students are asked to explore the properties possessed by a square, the data obtained will be filled into the table for the purpose of discussion in the next phase.

In explication phase, students express in their own words what they have discovered in the previous phase. The role of the teacher here is to introduce relevant geometrical terms. In this phase, students exchange their opinions about

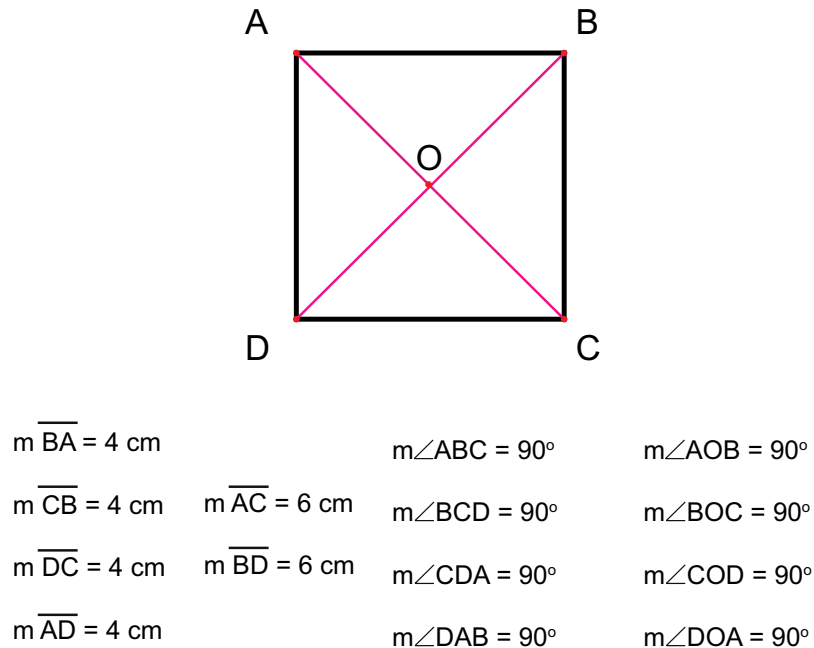


Figure 3. In the second phase, students investigate the properties of a square by using GSP.

the properties of rhombus [27]. In the topic of Triangles, students explain their experience with their classmates and teachers on the properties of each type of triangle by using their own words [16, 5]. In the topic of Circles, students discuss the relationship of the angles that they have explored in front of the class. Teachers then introduce the exact terminologies to the students [18]. In this research, students will explain their observations from the activities carried out earlier. With reference to the data derived from exploration using GSP, students can now explain the properties possessed by a square, rectangle, parallelogram, rhombus and kite. In Phase 4 which is free orientation, students will carry out more complex tasks; tasks that are more open-ended than in the guided orientation phase. The problems may be more complex and require more free exploration to find solutions. In this phase, a few edges and sides of rhombus are given in various positions and students are asked to build the whole figure of a rhombus [27]. In the free orientation phase in the study by Choi-Koh [5], students were given a triangle with two sides. They were then asked to put another side to make equilateral triangle, isosceles triangle, or right triangle. In this research, students are asked to connect the assigned dots to produce specified quadrilaterals. They can build a particular shape correctly if they understand the properties possessed by quadrilaterals. For

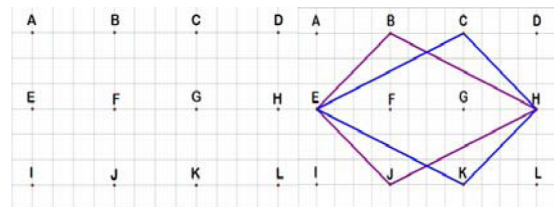


Figure 4. In the fourth phase, students connect the assigned dots to produce kites.

example, the diagram on the right shows kites constructed by connecting the points (Figure 4).

In the final phase; integration, students summarise and integrate what they have learned and develop a new network of objects and relations. This might be achieved in the form of discussions or an assignment. In the example given by Noraini [27], students summarise the properties of a rhombus in this phase. In the topic of Triangles, students summarise the various properties of triangles besides being able to differentiate the types of triangles based on their properties [16, 5]. In this research, the teacher will help students to summarise the concepts that they have explored and come to understand in this learning session. The students will be able to describe the properties possessed by the forms of the four sides of a square, rectangle, parallelogram, rhombus and kite.

3.2 Second Learning Session

The objective of this session is to assist students in increasing their geometric thinking from level 2 to level 3. Therefore, as shown in Figure 5, the activities in this session will be designed to help students strengthen their understanding on the properties of quadrilaterals and the relationships among them. Students will be able to verify these relationships by using non-formal deduction. In this learning session, students will again go through the phases in order to assist their movement from level 2; analysis to level 3; informal deduction.

In phase 1; information, students will reflect on the properties possessed by the quadrilaterals that they have produced in the previous session. They will now be asked to build quadrilaterals using the GSP. In guided orientation phase, the purpose of the activities is to help students identify the relationships among the quadrilaterals. Firstly, notes concerning the properties of quadrilaterals are provided in the GSP, and students will come to understand their properties in detail by clicking on the buttons provided. After analysing the quadrilaterals, they will then be asked to classify the quadrilaterals in terms of sides, angles and diagonals in the table. According to the data in the table, they are then asked to establish relationships among the quadrilaterals. Students and teachers will then discuss why a particular quadrilateral is distinct from other

quadrilaterals in the explication phase. In phase 4 which is free orientation, students are given a particular quadrilateral (for example, a rectangle). They are asked to find the value of its properties. They are then asked to determine, by dragging any vertices of the rectangle by using the GSP, why another quadrilateral (for example, a square) is a special case of the original quadrilateral (a rectangle). Next, they are asked to find the common properties possessed by these quadrilaterals. Finally, upon completion of the second learning session, in the integration phase, students will be able to summarise all the relationships among quadrilaterals. They can understand and will be able to distinguish the quadrilaterals by their definitions and classification.

4. Objectives of the Study

Based on the introduction and discussion of the Van Hiele's levels and phases of geometric learning, this study aims to improve the teaching and learning process of geometry topics. This study specifically aims at identifying the effectiveness of Van Hiele's phases of learning geometry using the Geometer's Sketchpad (GSP) on the level of geometric thinking in Form Two students. The topic involved was Form Two's Transformation, which included the Translation Concept, Reflection and Rotation, and Quadrilateral subtopics.

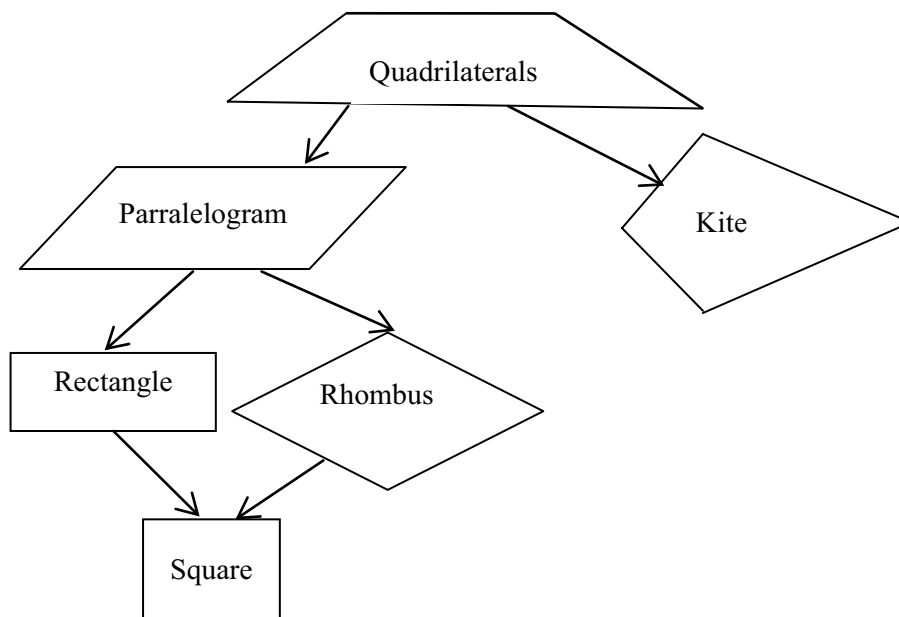


Figure 5. The relationships among the quadrilaterals.

5. Research Methodology

A quasi-experimental non-equivalent pretest-posttest control group design was used in this study. Ninety-four Form Two students were involved in this study, and they were divided into two groups, namely the control group and the treatment group. The students in the treatment group learned Form Two's Transformation topic based on Van Hiele's phases of learning geometry using the GSP software as an implementation medium. On the other hand, the control group learned the same topic using conventional methods. Van Hiele Geometry Test (UGVH) was given to both groups before and after the treatment. Ten students were randomly selected to further determine their initial and final levels of geometric thinking. The study was performed over six weeks.

6. Data Collection

The Van Hiele's Geometry Test (VHGT) and interview were used in this study as methods for data collection.

6.1 Van Hiele's Geometry Test (VHGT)

For quantitative data, the students' levels of geometric thinking were measured using VHGT, which was developed by the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) group from the University of Chicago [34]. However, the Malay version of VHGT was obtained from Tay [33]. Table 1 concludes the distribution of questions contained in the VHGT.

The marking is as follows:

A student was considered to have achieved a certain level in VHGT when he or she answered three out of five questions correctly. For example, as shown in Table 2, a student would be given one mark if he or she could answer at least three out of five questions correctly from questions

Table 1. Distribution of questions in VHGT

Van Hiele's levels of geometric thinking	Question number
Level 1: Visualisation	1 – 5
Level 2: Analysis	6 – 10
Level 3: Informal deduction	11 – 15
Level 4: Deduction	16 – 20
Level 5: Rigor	21 – 25

Source: Usiskin [34]

Table 2. Marking criteria in VHGT

Mark	Criteria of the items to be fulfilled	Van Hiele's levels of geometric thinking
1	1 – 5	1
2	6 – 10	2
4	11 – 15	3
8	16 – 20	4
16	21 – 25	5

Source: Usiskin [34]

Table 3. The weighted sum score for forced Van Hiele level

Forced Van Hiele level	Weighted sum score
0	0, 16, 2, 4, 8, 18, 20 or 24
1	1, 17, 5, 9, 21 or 25
2	3, 19, 11, or 27
3	7, 23, 22 or 6
4	15, 31, 29, 13, 14 or 30
No Fit	10, 12, 26 or 28

Source: Usiskin [34]

Classical Van Hiele level

Modified Van Hiele level

1 – 5, two marks for answering any three out of five questions from questions 6 – 10 correctly, and so on. The student's total mark in the VHGT was then calculated to determine the level of Van Hiele's geometric thinking possessed by the student. Forced Van Hiele level table was used as a reference to determine the student's level of geometric thinking.

Based on Table 3, a high mark does not mean that the student's level of geometric thinking also high. This is because, based on the characteristics of Van Hiele's model, students must go through the levels in the model sequentially and they must go through all levels in this model without leaving out any levels. For example, if the student can fulfil the criteria in level 1 and level 2, he or she will get 3 marks (1 + 2). If the student meets the criteria in level 1, level 2, and level 4, he or she will get 11 marks (1 + 2 + 8). However, based on the table, the student only achieves up to level 2 because he or she fulfils the criteria in level 1 and level 2 sequentially and skips level 3 even though he or she fulfils the criteria in level 4. As the data related to the students' levels of geometric thinking were ordinal scale data, Wilcoxon-t test for the design of repeated measurement was used in order to test the hypotheses. This Wilcoxon-t test has the same function as the t-test to do

Table 4. Descriptions of Van Hiele's level of geometric thinking

Level	Indication
Level 1: Recognition or visualization. Students identify shapes and other geometric configurations according to their appearance.	<ul style="list-style-type: none"> • The students identify instances of quadrilaterals by their appearance as a whole. • The students names or labels quadrilaterals and other geometric configurations and use standard or nonstandard names appropriately. • The students can construct, draw, or copy a quadrilateral. • The students verbally describe quadrilaterals by their appearance as a whole. • The students compare and sort quadrilaterals on the visual basis as a whole. • When the students sort quadrilaterals, they include imprecise visual information and irrelevant attributes while omitting relevant attributes. • The students do not consider the components or properties of quadrilaterals in order to identify or to name a quadrilateral. • The students are not able to formulate formal definitions of each type of quadrilaterals. The only definitions they can formulate consist of descriptions of physical attributes of the quadrilaterals.
Level 2: The students analyze figure in terms of their components and relationships between components, establishes properties of a class of figures empirically, and uses properties to solve problems.	<ul style="list-style-type: none"> • The students identify the components of quadrilaterals. • The students recalls and uses appropriate vocabulary for components and relationships. • The students compare two shapes according to relationships among their components. • The students sort quadrilaterals in different ways according to certain properties, including a sort of all instances of a class from non-instances. • The students intepret and use verbal description of a figure in terms of its properties and use the properties to draw or construct the figure. • The students discover properties of specific quadrilaterals empirically and generalize properties for that class of quadrilaterals. • The students describe a class of figures by means of their properties. • The students identify which properties are used to characterize one class of figures also apply to another class of figures and compares classes of figures according to their properties. • The students are not able to logically relate the properties to each other. • The students cannot logically classify quadrilaterals. They cannot explain subclass relationships.
Level 3: The students recognize subclass relationships between different types of quadrilaterals, formulate and use defintions, and give informal arguments that order previously discovered properties.	<ul style="list-style-type: none"> • The students identify different sets of properties that characterize a class of figures. • The students identify minimum sets of properties that can characterize a figure. • The students are able to formulate and use definitions for a class of quadrilaterals. • The students are able to accept and identify non-equivalent definitions of the same figures. • The students are able to logically classify quadrilaterals. • The students are able to provide informal arguments.

Source: Gutierrez et al. [14]

repeated measurement where the difference between these two tests is that the t-test is applied in repeated measurements to analyse two interval or ratio scale data groups, while Wilcoxon-t test analyses two ordinal scale data groups.

6.2 Interview

An interview method was conducted to further identify the differences in the students' levels of geometric thinking towards the geometric concepts. The interview

method has been demonstrated by many researchers to be the most effective method to determine the students' level of geometric thinking. It provides in-depth information about how the students think compares to other methods [14, 33]. According to Atebe [1], the interview method is used to identify the levels of geometric thinking, as tests using pen and paper can not provide sufficient information about their levels. By using an interview method, the students have an opportunity to express their thoughts interactively during the interview sessions. Furthermore, according to Dindyal [13], the combination of quantitative and qualitative

methods such that in interview can provide more accurate information about the level of geometric thinking. Other than that, by using the interview method, researchers can compare the answers given by the students on the same tasks [3]. The items used in the interview were those found in Van Hiele Geometry Test (VHGT), which was developed in a study by Usiskin [34]. The researcher has obtained permission from the developer to use the instrument. The Malay version of the items were obtained from a study by Tay [33]. However, the researcher only used the items from the first level to the third level, as many previous studies have shown that secondary school students can perform well only up to Van Hiele's third level of geometric thinking.

To identify the degree and level of geometric thinking of the students involved in this study, the researcher used the method proposed by Gutierrez et al. [14]. The pre- and post-interviews from the students were transcribed first. Based on the answers given in the interviews, their level of geometric thinking was determined and the vectors were assigned based on the description shown in Table 4. As proposed by Gutierrez et al. [14], answers from the students who were at the transition level, which is the level between two levels, were determined as being at the higher level. This was because those answers indicated that the students, to a certain degree of acquisition, came very close to achieving that level.

Next, referring to Table 5, each answer was assigned to one of the eight types of answers, depending on the mathematical accuracy and degree of reasoning. Finally,

the degree of acquisition for a given level by the students was determined by a vector quantity (level, type) suitable for all the items answered at that particular level.

After the suitable vector quantity (level, type) for all the items answered in that particular level had been identified, the student's Van Hiele's degree of acquisition was determined by calculating the mean value for each level and for each student based on the weight value assigned to each type of answer. The weight values are shown in Table 6.

After the mean value for each level had been obtained, the student's Van Hiele's degree of acquisition value was determined based on Figure 6.

7. Data Analysis

After all the test and interview have been conducted, the data were mainly utilized to analyze on students' Van Hiele level.

7.1 Initial Levels of Students' Geometric Thinking

H_{02} : *There is no significant difference between the students' initial levels of geometric thinking in the treatment group and control group.*

To test the above hypothesis, the Wilcoxon-t test for the design of matching samples to make comparison between two matching group samples in two different situations was used.

Table 5. The descriptions of each type of answer

Type of answer	Descriptions
Type 0	No reply or answer that cannot be codified.
Type 1	Answers that indicate the learner has not attained a given level but that give no information about any lower level.
Type 2	Wrong and insufficiently worked out answers that give some indication of a given level of reasoning; answers that contain incorrect and reduced explanations, reasoning processes, or results.
Type 3	Correct but insufficiently worked out answers that give some indication of a given level of reasoning; answers that contain very few explanations, inchoate reasoning processes, or very incomplete results.
Type 4	Correct or incorrect answers that clearly reflect characteristic features of two consecutive van Hiele levels and that contain clear reasoning processes and sufficient justifications.
Type 5	Incorrect answers that clearly reflect a level of reasoning; answers that present reasoning processes that are complete but incorrect or answers that present correct reasoning processes that do not lead to the solution of the stated problem.
Type 6	Correct answers that clearly reflect a given level of reasoning but that are incomplete or insufficiently justified
Type 7	Correct, complete and sufficiently justified answers that clearly reflect a given level of reasoning.

Source: Gutierrez et al. [14]

Table 6. Weights of different types of answers

Type	0	1	2	3	4	5	6	7
Weight	0	0	20	25	50	75	80	100

Source: Gutierrez et al. [14]

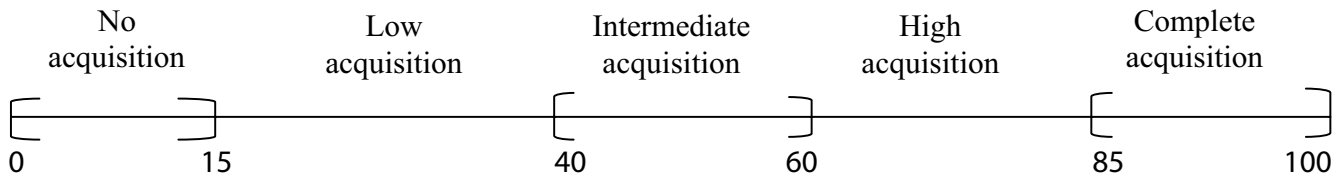


Figure 6. Degrees of acquisition of a Van Hiele level.

Source: Gutierrez et al. [14]

Based on Table 7, the significant value .439 is more than .05. The result of Wilcoxon-t test is significant ($T = 48.00, p > 0.05$), which shows that there is no significant difference between the initial levels of geometric thinking of the two groups. The descriptive statistics of initial levels of geometric thinking for control and treatment groups are shown in Table 8.

As mentioned earlier, ten students were randomly selected, with five students from both the control group and the treatment group. Student A, B, C, D and E were in control group while student F, G, H, I and J were in the treatment group.

Based on Figures 7 and 8, it can be seen that both groups are balanced for the acquisition of geometric thinking. The majority of the students attained a complete acquisition of the first level of geometric thinking, which is visualisation. Almost all the students in both groups

Table 7. Wilcoxon-t statistics test

	Treatment group – Control group
Z	-0.775(a)
Asymp. Sig. (2-tailed)	.439

Table 8. Descriptive statistics of initial levels of geometric thinking for control and treatment groups

	N	Mean	Std. Deviation	Minimum	Maximum
Control group	47	1.17	.524	0	2
Treatment group	47	1.23	.428	1	2

showed a low acquisition of second level, while almost all failed to reach the third level of informal deduction.

7.2 Final Levels of Students’ Geometric Thinking

H_{02} : There is no significant difference between the students’ final levels of geometric thinking in the treatment group and control group.

To test the hypothesis, the Wilcoxon-t test for the design of matching samples to make comparison between two matching group samples in two different situations was used.

Based on Table 9, the significant value .00 is less than .05. The result of Wilcoxon-t test is significant ($T = 34.50, p < 0.05$), which shows that there is a significant difference between the final levels of geometric thinking of the two groups. The descriptive statistics of final levels of geometric thinking for control and treatment groups are shown in Table 10. This result is supported by the Boxplot graph median value for both the ordinal scores of the two groups (Figure 9), which clearly shows the treatment group’s final levels of geometric thinking is higher than the control group’s final levels of geometric thinking.

Based on Figure 10 and 11, it can be seen that there is a significant difference in the final levels of geometric

Table 9. Wilcoxon-t statistics test Abdullah and Zakaria [40]

	Treatment group – Control group
Z	-4.388(a)
Asymp. Sig. (2-tailed)	.000

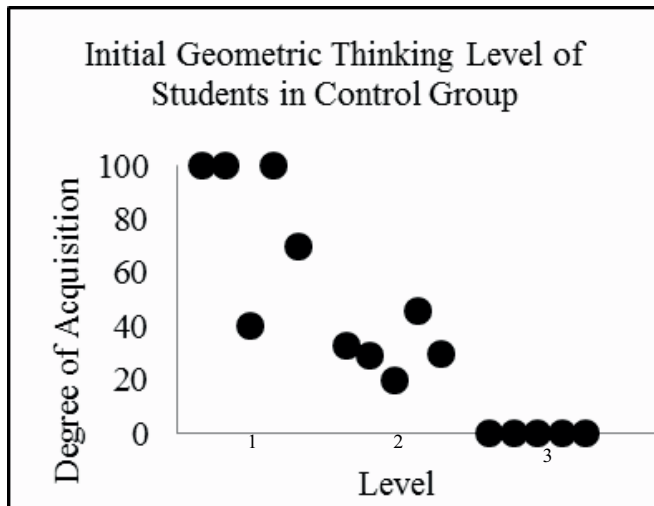


Figure 7. Scatter plot for the degree of acquisition of the initial geometric thinking level for the students in the control group.

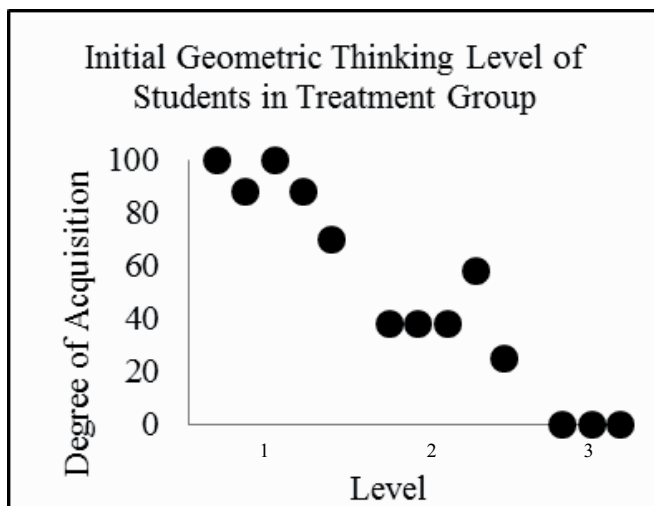


Figure 8. Scatter plot for the degree of acquisition of the initial geometric thinking level for the students in the treatment group.

To determine further the initial level of geometric thinking of the control group, qualitative data were analysed. It can be summarised that student A and B attained a complete acquisition on the visualisation level. However, they showed low acquisition on the analysis level, and they did not reach the informal deduction level. Student C attained an intermediate acquisition level for the first level, low level for the second level and did not score on the third level. Student D showed complete acquisition of the first level, an intermediate acquisition level for the second and did not score on the third. Student E attained a high acquisition rating for the first level, a low rating for second level and did not reach the third level.

In details, for the students in the treatment group, it was found that four students, namely Student F, G, H, and I, showed complete acquisition of visualisation level. Student J showed a high acquisition on the first level, while Student F showed a low acquisition on the analysis level. As for Student F, G, H and J, they were low on the analysis level, and they did not reach the informal deduction level. However, Student I managed to show an intermediate acquisition rating at the second level and a high acquisition rating at the third.

Table 10. Descriptive statistics of final levels of geometric thinking for control and treatment groups Abdullah and Zakaria [40]

	N	Mean	Std. Deviation	Minimum	Maximum
Control group	47	1.51	.547	1	3
Treatment group	47	2.23	.698	1	3

thinking between the two groups. Students in the control group showed improvement in the first and second levels, although there were two students who showed degradation from a complete acquisition after first level. However, all the students showed improvement in the second, analysis level, at which they improved from a low and an intermediate acquisition level to an intermediate and a high acquisition level. None of the students in the control group attained the

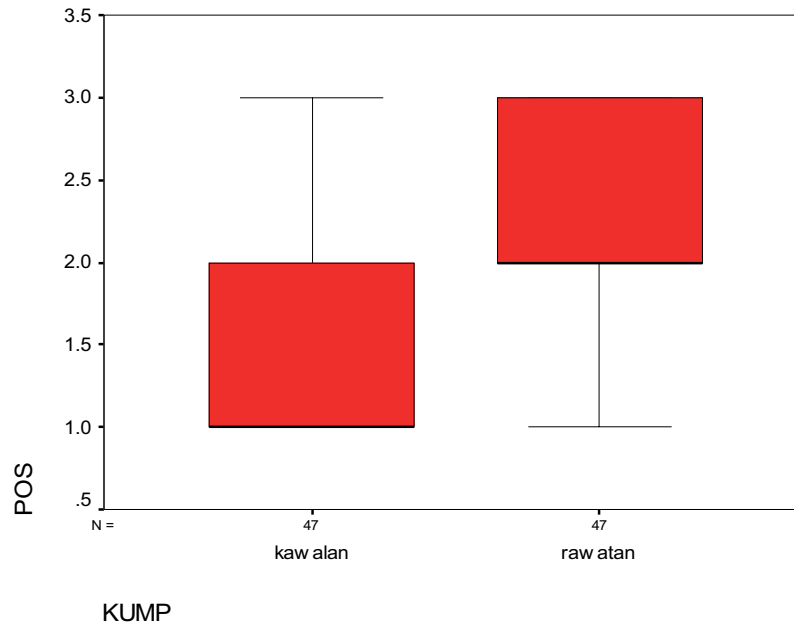


Figure 9. Boxplot graph of the final levels of geometric thinking for control and treatment groups Abdullah and Zakaria [40].

third level, which is informal deduction. On the other hand, the students in the treatment group showed improvement in all the three levels, with all of them attaining complete acquisition of visualisation level. One student attained an intermediate acquisition, while another one scored a high acquisition of second level. Three other students attained a complete second level. As for the third level, only one student did not manage to score that level. The rest of the students managed to attain a complete and a high acquisition rate for the third level of geometric thinking.

8. Discussion

Based on the analyses, most of the students' initial levels of geometric thinking were at the first level, which is visualisation. This is parallel to the finding by Razananadiah [27] that most students only achieved the first level at the beginning of their school education. This finding is also in line with the finding obtained in the study conducted by Chong [6] and Noraini [24] who found that the majority of the students achieved the visualisation level of geometric thinking before intervention was introduced. This was highly probable because the visualisation level is the most basic level and does not involve the argumentative ability in students but is more about their perspective [19]. At this level, students recognise and identify certain geometric shapes based on the overall entity of the objects [8, 15, 25].

This can be with the assistance of the lesson about the essentials of shapes, which the students have been exposed to in primary school [11]. The findings also revealed that the final students' levels of geometric thinking in the treatment group were better than the levels of geometric thinking for students from the control group. Therefore, this means that the implementation of Van Hiele's phases of learning geometry with assistance from GSP software assisted students in achieving better levels of geometric thinking as compared to those students who learned the topics conventionally. The findings of this study are in accordance with previous studies that were conducted by Teppo [33], Matthews [20], Wu [38] and Craft [7]. In a Malaysian context, these findings are in line with the studies conducted by Tay [33] who focused on application of manipulative materials in the phase-based activities and Chew [4] who focused on other geometry topics. This study has also showed that improvement from one level of geometric thinking to a higher level of geometric thinking depends on the lesson taken by the students and not on their maturity [36]. The method and learning organisation and also the contents and teaching aids used are the important elements of the pedagogy [36].

9. Conclusion

The aim of this study was to identify the effectiveness of Van Hiele's phases of learning geometry in the learning

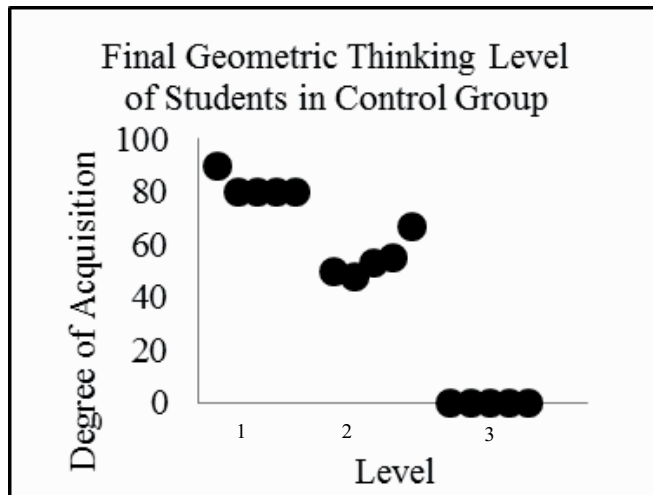


Figure 10. Scatter plot for the degree of acquisition of the final geometric thinking level for the students in the control group.

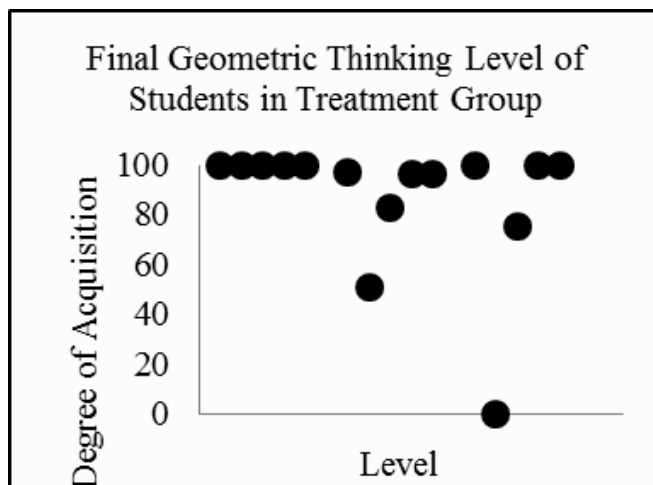


Figure 11. Scatter plot for the degree of acquisition of the final geometric thinking level for the students in treatment group.

To further determine the initial level of geometric thinking of the control group, qualitative data were analysed. It can be summarised that almost all the students, namely Student B, C, D, and E, attained a high acquisition for first level thinking, with only Student A attaining complete acquisition of first level. Student A, B, C, and D showed an intermediate acquisition for second level. One student showed a high acquisition rating for second level. However, none of the students in the control group scored on the third level.

In details, for the students in treatment group, Student F, I and J managed to reach the three levels of visualisation, analysis and informal deduction. Student G showed a complete acquisition of first level, an intermediate acquisition of second level, and did not each the third level. Student H managed to attain a complete acquisition of first level, and a level high acquisition for the second and third level of geometric thinking.

of Form Two's Transformation topic that consists of the Concepts of Transformation (Translation, Reflection, and Rotation) and Quadrilaterals subtopics in order to assist the students in enhancing their levels of thinking to higher levels. The phases involved were information, guided orientation, explication, free orientation, and integration. Geometer's Sketchpad (GSP) software was used as a medium to implement the activities. The students in the treatment group learned the Transformation topic based on the activities developed from the Van Hiele's phases of learning geometry by using the GSP software. Meanwhile,

the students in the control group learned the same topic using the conventional approach. The students' initial and final levels of geometric thinking in both groups were identified quantitatively and qualitatively. It was found that students in the treatment group showed a better increment of geometric thinking levels compares to students in the control group. Therefore, in accordance with the national education transformation concept as stressed by Malaysia's Ministry of Education (MOE), teachers should introduce new approaches in their teaching and learning practices in the topics of geometry. One new approach that can be

implemented is delivering the contents of geometry topics based on Van Hiele's phases of learning geometry. The GSP software, the license of which has been bought by the MOE to be used in schools, can be beneficial, besides its advantages in the teaching and learning process that have certainly been proven by previous studies. This is important in the context of learning geometry in Malaysia because the geometry topics comprise about 40% of the Mathematics topics taught in secondary schools.

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