Evaluating the efficiency of decision making units in the presence of flexible and negative data

[1]S. Kordrostami, [2]M. Jahani Sayyad Noveiri

[1, 2] *Department of Applied Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran*
[1]sohrabkordrostami@gmail.com, [2] monirehjahani@yahoo.com,

Abstract

Looking into the literature reveals that conventional data envelopment analysis (DEA) models assume non-negative inputs and outputs in which the status of each measure is known as either input or output. However, occasions arise where a measure can play either input or output roles and can take negative values. Emrouznejad et al., (2010a) [Emrouznejad A, Anouze A.L and Thanassoulis E (2010a)A semi-oriented radial measure for measuring the efficiency of decision making units with negative data, using DEA. European Journal of Operational Research, 200, 297-304.] introduced a Semi-Oriented Radial Measure (SORM) for modeling DEA with negative data. The current paper presents a modification of the SORM model to accommodate such flexible measures, and indeed a model is proposed to evaluate the efficiency of decision making units (DMUs) where flexible and negative data exist. Several examples illustrate the proposed approach.

Keywords: Data envelopment analysis, Negative data in DEA, Flexible measure, Efficiency.

1. Introduction

The realm of Data envelopment analysis (DEA), initially proposed by Charnes et al. (1978), provides a relative efficiency measure for peer decision making units (DMUs) with multiple inputs and outputs. After the initial works of Charnes et al. (1978), a number of scholars have proposed different DEA models which are widespread in the literature of Cooper et al. (2000). If we take a look at the conventional application of DEA, we notice that the status of performance measures from the view point of input or output is specified. However, conditions arise in which we encounter measures, named as "flexible measures" by Cook and Zhu (2007), that treat as both an input and output; moreover, finding the appropriate status of them is difficult. By having a more exact look at Cook and Zhu's work, we find that they actually proposed a DEA model with the function of classifying a measure into an input or output and evaluating the performance of DMUs. The next scholar to work in this realm was Toloo (2009) who introduced the modified model in order to evaluate the efficiency of DMUs in the presence of flexible measures. Likewise, Amirteimoori et al. (2011) proposed an alternative model of this, commented on Toloo’s model (Amirteimoori et al., 2012). Additionally, Toloo (2012) considered alternative solutions for classifying inputs and outputs in data envelopment analysis. Taking a glance at the literature of DEA reveals that there have been various approaches for dealing with negative data until now. Emrouznejad et al. (2010a) for instance suggested the Semi-Oriented Radial Measure (SORM) to handle variables taking both positive and negative values over the units. We can also refer to papers Scheel (2001), Portela et al.(2004), Sharp et al.(2006), Kazemi Matin et al. (2011), Emrouznejad et al.(2010b) and Hadad et al.(2012); they all provided models to investigate negative data. In our study here, we have tried to modify the SORM model to accommodate such flexible measures as mentioned above. In order to explain the possible application of our proposed model, assume a factor such as the number of worthwhile customers that can be considered as an input and an output to evaluate bank branch operations. Cook and Zhu (2007) declared "from one prospective, such a measuring play the role of proxy for future investment, hence can reasonably be classified as an output. On the other hand, it can legitimately be considered as an environmental input that aids the branch in generating its existing investment portfolio". The important point that we should consider here is that growth in number of customers can take positive and negative values making growth in the number of customers as a flexible and negative measure. Flexible and negative data are encountered in many real world situations. So here we aim to propose a model that evaluates the performance of DMUs in the presence of flexible and negative data. We present an approach in which each one of the flexible variables is treated as either input or output to maximize the technical efficiency of the DMU under evaluation while measures can take negative and/or positive values. The rest of this paper is organized as follows. Section 2 gives a brief explanation of the SORM model. Section 3 introduces the proposed model. Numerical examples are provided in section 4. Finally conclusions are given in section 5.

2. A Semi-Oriented Radial Measure (SORM)

Consider a set of n DMU_j $(j=1,\ldots,n)$, consuming m inputs, x_{ij} $(i=1,\ldots,m)$ to produce s outputs, y_{rj} $(r=1,\ldots,s)$. In (1) and (2) below we see the Banker et al.'s (1984) input and output oriented DEA models for evaluating the technical efficiency of DMU_o under the assumption of variable returns to scale (VRS). The performance of the models (1) and (2) are pictured as the optimal values h and $1/h$ respectively.

$$\begin{aligned} &Min \;\; h \\ &s.t. \;\; \sum_j \lambda_j x_{ij} \le h x_{ij_o};\ \forall i \\ &\sum_j \lambda_j y_{rj} \ge y_{rj_o};\ \forall r \\ &\sum_j \lambda_j = 1. \quad \lambda_j \ge 0;\ \forall j. \end{aligned} \tag{1}$$

$$\begin{aligned} &Max \;\; h \\ &s.t. \;\; \sum_j \lambda_j x_{ij} \le x_{ij_o};\ \forall i \\ &\sum_j \lambda_j y_{rj} \ge h y_{rj_o};\ \forall r \\ &\sum_j \lambda_j = 1. \quad \lambda_j \ge 0;\ \forall j. \end{aligned} \tag{2}$$

Emrouznejad et al. (2010a), by using the absolute value definition, proposed a model to deal with negative data in which they utilized a partitioning approach in modeling negative data thus suggesting a Semi-Oriented Radial measure (SORM) for performance evaluation of the observed production units. Let us bring forward a brief description of SORM here; for this let the input variable $x_i, i \in I$ and the output variable $y_r, r \in R$ are positive for all DMUs. Further assume that the input variable $x_\ell, \ell \in L$ is positive for some DMUs and negative for others and $y_k, k \in K$ are outputs which take positive values for some DMUs and negative for others. It is clear that

$I \bigcup L = \{1,\ldots,m\},\ I \bigcap L = \varnothing,\ R \bigcup K =$

$\{1,\ldots,s\}, R \bigcap K = \varnothing.$ So we can write

$$x_{\ell j} = x^1_{\ell j} - x^2_{\ell j}; x^1_{\ell j}, x^2_{\ell j} \ge 0, y_{kj} = y^1_{kj} - y^2_{kj}; y^1_{kj}, y^2_{kj} \ge 0,$$

where:

$$x^1_{\ell j} = \begin{cases} x_{\ell j};\ if \;\; x_{\ell j} \ge 0 \\ 0;\ \ if \;\; x_{\ell j} < 0 \end{cases} \qquad x^2_{\ell j} = \begin{cases} 0; \qquad if \;\; x_{\ell j} \ge 0 \\ -x_{\ell j};\ \ if \;\; x_{\ell j} < 0 \end{cases}$$

$$y^1_{kj} = \begin{cases} y_{kj};\ if \;\; y_{kj} \ge 0 \\ 0;\ \ if \;\; y_{kj} < 0 \end{cases} \qquad y^2_{kj} = \begin{cases} 0; \qquad if \;\; y_{kj} \ge 0 \\ -y_{kj};\ \ if \;\; y_{kj} < 0 \end{cases}$$

Having the above-mentioned notations in mind, the SORM model in its output orientation could be stated as the following LP problem.

$$\begin{aligned} &Max \;\; h \\ &s.t. \;\; \sum_j \lambda_j x_{ij} \le x_{ij_o};\ \forall i \in I, \\ &\sum_j \lambda_j x^1_{lj} \le x^1_{lj_o};\ \forall l \in L, \\ &\sum_j \lambda_j x^2_{lj} \ge x^2_{lj_o};\ \forall l \in L, \\ &\sum_j \lambda_j y_{rj} \ge h y_{rj_o};\ \forall r \in R, \\ &\sum_j \lambda_j y^1_{kj} \ge h y^1_{kj_o};\ \forall k \in K, \\ &\sum_j \lambda_j y^2_{kj} \le h y^2_{kj_o};\ \forall k \in K, \\ &\sum_j \lambda_j = 1. \quad \lambda_j \ge 0;\ \forall j. \end{aligned} \tag{3}$$

In the presence of negative data, the optimal solution of this model, i.e. $1/h^*$, represents the SORM efficiency of DMU_o.

3. A SORM model in the presence of flexible measures

Consider the output oriented VRS SORM model (3). Assume that there are P flexible measures w_{tj} $(t = 1,\ldots,P)$ in which the status of input/output is unknown. Here we are going to delineate different cases of the model which are possible:

The first assumption that we make is that the flexible measure $w_t,\ t \in T$ is positive for all DMUs. So we have the possibility of adding more constraints to the model (3).

$$\begin{aligned} &\sum_j \lambda_j w_{tj} \le w_{tj_o};\ \forall t \in T, \\ &or \\ &\sum_j \lambda_j w_{tj} \ge h w_{tj_o};\ \forall t \in T. \end{aligned} \tag{4}$$

As seen in the following part, we utilize a procedure for solving problems with either/or constraints. (For further details see Chinneck (2004), Amirteimoori (2011), and Toloo (2012))

$$\begin{aligned} &\sum_j \lambda_j w_{tj} \le w_{tj_o} + M(1-z_t);\ \forall t \in T, \\ -&\sum_j \lambda_j w_{tj} \le -h w_{tj_o} + M z_t;\ \forall t \in T. \end{aligned} \tag{5}$$

Here M and z_t are a large positive number and a binary variable respectively. Also, t is an output factor where $z_t = 0$ and it is an input factor where $z_t = 1$.

Therefore, model (3) is reformulated to the following mixed integer linear program:

$$\begin{aligned} Max\ \ & h \\ s.t.\ \ & \sum_j \lambda_j x_{ij} \le x_{ij_o};\ \forall i \in I, \\ & \sum_j \lambda_j x^1_{lj} \le x^1_{lj_o};\ \forall l \in L, \\ & \sum_j \lambda_j x^2_{lj} \ge x^2_{lj_o};\ \forall l \in L, \\ & \sum_j \lambda_j y_{rj} \ge h y_{rj_o};\ \forall r \in R, \\ & \sum_j \lambda_j y^1_{kj} \ge h y^1_{kj_o};\ \forall k \in K, \\ & \sum_j \lambda_j y^2_{kj} \le h y^2_{kj_o};\ \forall k \in K, \\ & \sum_j \lambda_j = 1; \quad \lambda_j \ge 0;\ \forall j, \\ & \sum_j \lambda_j w_{tj} \le w_{tj_o} + M(1-z_t);\ \forall t \in T, \\ & -\sum_j \lambda_j w_{tj} \le -h w_{tj_o} + M z_t;\ \forall t \in T;\ z_t \in \{0,1\}. \end{aligned} \tag{6}$$

Next we consider a flexible variable $w_t, t \in T$ which is positive for some DMUs and negative for others. By defining the two variables w^1_t and w^2_t which for the jth DMU take values w^1_{tj} and w^2_{tj} we have:

$$w^1_{tj} = \begin{cases} w_{tj};\ if & w_{tj} \ge 0 \\ 0;\ \ if & w_{tj} < 0 \end{cases} \qquad w^2_{tj} = \begin{cases} 0; & if\ \ w_{tj} \ge 0 \\ -w_{tj}; & if\ \ w_{tj} < 0 \end{cases}$$

The point that should be noticed here is that $w^1_{tj} \ge 0$ and $w^2_{tj} \ge 0$ while $w_{tj} = w^1_{tj} - w^2_{tj}$ for all j. To determine the status of these variables the following constraints are applied as follows:

$$\begin{cases} \sum_j \lambda_j w^1_{tj} \le w^1_{tj_o};\ \forall t \in T, \\ or \\ \sum_j \lambda_j w^1_{tj} \ge h w^1_{tj_o};\ \forall t \in T, \end{cases} \qquad \begin{cases} \sum_j \lambda_j w^2_{tj} \ge w^2_{tj_o};\ \forall t \in T, \\ or \\ \sum_j \lambda_j w^2_{tj} \le h w^2_{tj_o};\ \forall t \in T. \end{cases} \tag{7}$$

By imposing the following constraints they can be pictured in model (3) as seen below:

$$\begin{cases} \sum_j \lambda_j w^1_{tj} \le w^1_{tj_o} + M(1-z_t);\ \forall t \in T, \\ -\sum_j \lambda_j w^1_{tj} \le -h w^1_{tj_o} + M z_t;\ \forall t \in T, \end{cases} \qquad \begin{cases} -\sum_j \lambda_j w^2_{tj} \le -w^2_{tj_o} + M(1-\bar{z}_t);\ \forall t \in T, \\ \sum_j \lambda_j w^2_{tj} \le h w^2_{tj_o} + M \bar{z}_t;\ \forall t \in T. \end{cases} \tag{8}$$

In this case model (3) is restated as follow:

$$\begin{aligned} Max\ \ & h \\ s.t.\ \ & \sum_j \lambda_j x_{ij} \le x_{ij_o};\ \forall i \in I, \\ & \sum_j \lambda_j x^1_{lj} \le x^1_{lj_o};\ \forall l \in L, \\ & \sum_j \lambda_j x^2_{lj} \ge x^2_{lj_o};\ \forall l \in L, \\ & \sum_j \lambda_j y_{rj} \ge h y_{rj_o};\ \forall r \in R, \\ & \sum_j \lambda_j y^1_{kj} \ge h y^1_{kj_o};\ \forall k \in K, \\ & \sum_j \lambda_j y^2_{kj} \le h y^2_{kj_o};\ \forall k \in K, \\ & \sum_j \lambda_j = 1; \quad \lambda_j \ge 0;\ \forall j, \\ & \sum_j \lambda_j w^1_{tj} \le w^1_{tj_o} + M(1-z_t);\ \forall t \in T, \\ & -\sum_j \lambda_j w^1_{tj} \le -h w^1_{tj_o} + M z_t;\ \forall t \in T, \\ & -\sum_j \lambda_j w^2_{tj} \le -w^2_{tj_o} + M(1-\bar{z}_t);\ \forall t \in T, \\ & \sum_j \lambda_j w^2_{tj} \le h w^2_{tj_o} + M \bar{z}_t;\ \forall t \in T \\ & \bar{z}_t, z_t \in \{0,1\}. \end{aligned} \tag{9}$$

A special case of the previous one arises in which all of variables are negative, so the following constraints are added:

$$-\sum_j \lambda_j w_{tj} \le -w_{tj_o} + M(1-z_t);\ \forall t \in T,$$
$$\sum_j \lambda_j w_{tj} \le h w_{tj_o} + M z_t;\ \forall t \in T. \qquad (10)$$

Until this point we have seen that the above models can be solved to evaluate efficiency where flexible and negative data is found. Afterwards, the majority choice among the DMUs can be used to deciding the appropriate status of a flexible variable.

As you may have noticed the preceding models are illustrated in an output orientation. These models can be reformulated in an input orientation. Now model (9) is modified to evaluate efficiency in an input orientation as follows:

$$\begin{aligned}
&Min\ \ h\\
&s.t.\quad \sum_j \lambda_j x_{ij} \le h x_{ij_o};\ \forall i \in I,\\
&\sum_j \lambda_j x^1_{lj} \le h x^1_{lj_o};\ \forall l \in L,\\
&\sum_j \lambda_j x^2_{lj} \ge h x^2_{lj_o};\ \forall l \in L,\\
&\sum_j \lambda_j y_{rj} \ge y_{rj_o};\ \forall r \in R,\\
&\sum_j \lambda_j y^1_{kj} \ge y^1_{kj_o};\ \forall k \in K,\\
&\sum_j \lambda_j y^2_{kj} \le y^2_{kj_o};\ \forall k \in K,\\
&\sum_j \lambda_j = 1; \quad \lambda_j \ge 0;\ \forall j, \qquad (11)\\
&\sum_j \lambda_j w^1_{tj} \le h w^1_{tj_o} + M(1-z_t);\ \forall t \in T,\\
&-\sum_j \lambda_j w^1_{tj} \le -w^1_{tj_o} + M z_t;\ \forall t \in T,\\
&-\sum_j \lambda_j w^2_{tj} \le -h w^2_{tj_o} + M(1-\bar{z}_t);\ \forall t \in T,\\
&\sum_j \lambda_j w^2_{tj} \le w^2_{tj_o} + M\bar{z}_t;\ \forall t \in T \quad \bar{z}_t, z_t \in \{0,1\}.
\end{aligned}$$

According to Emrouznejad et al. (2010a) feasible solution in model (3) will also be feasible in model (2), so we see that in model (3) the axioms for creating the production possibility set (PPS) in DEA under VRS are valid. Now assume that T denotes the production possibility set. Additionally we can reach an extension (development) of the VRS technology, by considering a flexible measure w in the following way:

1. $(x_j, y_j, w_j) \in T$

2. Convexity:

$$(x,y,w),(x',y',w') \in T,\ (\bar{x},\bar{y},\bar{w}) =$$

$$\lambda(x,y,w)+(1-\lambda)(x',y',w') \in T,$$

$$0 \le \lambda \le 1 \Rightarrow (\bar{x},\bar{y},\bar{w}) \in T.$$

3. Free disposability:

$$(x,y,w) \in T,\ x' \ge x,\ y' \le y, and$$
$$(either\, w' \ge w\ \ or\ w' \le w) \Rightarrow (x',y',w') \in T$$

4. Minimum extrapolation principle: the DEA PPS is the intersection of all sets that contain all observed DMUS and satisfy the maintained set of axioms.

Under the above axioms, the VRS technology is defined as follows:

$$T = \left\{ (x,y,w) \middle| x \ge \sum_{j=1}^n \lambda_j x_j,\ y \le \sum_{j=1}^n \lambda_j y_j, (either\ \ w \ge \sum_{j=1}^n \lambda_j w_j\ or\ w \le \sum_{j=1}^n \lambda_j w_j); \sum_{j=1}^n \lambda_j = 1, \lambda \ge 0 \right\}$$

Note Amirteimoori et al. (2011) stated an axiomatic foundation where flexible measures present and under constant returns to scale (CRS). Readers can refer to it for more information.

4. Numerical examples

In this section we will apply our proposed model to Emrouznejad et al.'s data set (2010a), which it had been utilized by Sharp et al. (2006) first, and also in the data set of a couple of Iranian banks.

Example 1 Assume that we have thirteen DMUs in Table 1. Cost and effluent are considered as input variables while output variables are saleable and CO_2. Furthermore, Methane is supposed to be a flexible measure. The results of model (3) are reported in the second and third columns of Table 2, where the second column shows the optimal value $1/h^*$ when Methane is considered as an input, and the third column denotes optimal value $1/h^*$ when it is assumed as an output. The fourth column displays the efficiency to model (9), where Methane is assumed as a flexible measure. The optimal z is depicted in the fifth column. As can be seen, the efficiency scores of DMU3, DMU7, DMU8, DMU11, and DMU13 when Methane is assumed as an input are equal with when it is supposed as an output. We reach the conclusion that these DMUs must not be taken into account for classifying inputs and outputs. The results show all remaining DMUs treat the flexible measure as an output.

Table 1. *National effluent processing system*

DMU	Cost	Effluent	Saleable	CO_2	Methane
DMU1	1.03	-0.05	0.56	-0.09	-0.44
DMU2	1.75	-0.17	0.74	-0.24	-0.31
DMU3	1.44	-0.56	1.37	-0.35	-0.21
DMU4	10.8	-0.22	5.61	-0.98	-3.79
DMU5	1.3	-0.07	0.49	-1.08	-0.34
DMU6	1.98	-0.1	1.61	-0.44	-0.34
DMU7	0.97	-0.17	0.82	-0.08	-0.43
DMU8	9.82	-2.32	5.61	-1.42	-1.94
DMU9	1.59	0	0.52	0	-0.37
DMU10	5.96	-0.15	2.14	-0.52	-0.18
DMU11	1.29	-0.11	0.57	0	-0.24
DMU12	2.38	-0.25	0.57	-0.67	-0.43
DMU13	10.3	-0.16	9.56	-0.58	0

Table 2. *Results of models (3) and (9) where Methane is assumed as a flexible measure*

DMU	Input	Output	Flexible	Z
DMU1	0.6501105	0.6290495	0.6290495	0
DMU2	0.4633276	0.4467078	0.4467078	0
DMU3	1	1	1	0 or 1
DMU4	1	0.593648	0.593648	0
DMU5	0.4186202	0.4062398	0.4062398	0
DMU6	0.8993615	0.8613264	0.8613264	0
DMU7	1	1	1	0 or 1
DMU8	1	1	1	0 or 1
DMU9	1	0.9122423	0.9122423	0
DMU10	0.3886967	0.3857132	0.3857132	0
DMU11	1	1	1	0 or 1
DMU12	0.279275	0.254589	0.254589	0
DMU13	1	1	1	0 or 1

In a second example we assume that cost is an input measure; saleable, CO2 and Methane are output variables and effluent is a flexible measure respectively. The second and third columns of the Table 3 show the optimal value $1/h^*$ from model (3) when effluent is assumed as an input and an output. The fourth column depicts the results arising from model (9) when effluent is assumed as a flexible measure. In this case, as can be seen in the fifth column; 6 out of 13 DMUs design effluent as an input or an output and 5 out of the 7 remaining DMUs treat it as an input.

Table 3. *Results of models (3) and (9) where effluent is assumed as a flexible measure*

DMU	Input	Output	Flexible	Z
DMU1	0.6290495	1	0.6290495	1
DMU2	0.4467078	0.4605324	0.4467078	1
DMU3	1	1	1	0 or 1
DMU4	0.593648	0.58682	0.58682	0
DMU5	0.4062398	0.4449784	0.4062398	1
DMU6	0.8613264	1	0.8613264	1
DMU7	1	1	1	0 or 1
DMU8	1	0.6153846	0.6153846	0
DMU9	0.9122423	1	0.9122423	1
DMU10	0.3857132	0.3857132	0.3857132	0 or 1
DMU11	1	1	1	0 or 1
DMU12	0.254589	0.254589	0.254589	0 or 1
DMU13	1	1	1	0 or 1

All remaining DMUs have considered the flexible measure as an output in the first part of example 1 provided that we apply the majority choice among the DMUs to determine the overall input/output status of every flexible measure. Besides, in the second part of example 1, the most DMUs treat the flexible measure as an input. These results are compatible with the main status of these variables.

Example 2 Now we apply our proposed model to 20 Iranian commercial bank branches for the month of July 2012. This data set consists of two inputs, two outputs, and a flexible measure. Inputs include current cost(t) and the number of staff(t) while resources(t) and loans(t) comprise outputs; in addition, Δ (the Greek Δ indicates the change in number of clients from month $t-1$ to month t and t denotes time period) number of clients is assumed as a flexible measure. It is evident that such variable can take positive and negative values. The amount of inputs and outputs are pictured in Table 4.

Table 4. *Data of the bank branches in month t*

#DMU	Staff	Current cost	Client	Resource	Loan
1	10	9739	50	107056	47739
2	6	9248	10	67709	47214
3	5	5744	20	41278	31623
4	9	8128	35	62499	28150
5	8	6190	-20	101870	61918
6	6	6936	5	55104	12222

#DMU	Staff	Current cost	Client	Resource	Loan
7	10	5556	53	60709	20457
8	5	5843	20	60165	24520
9	4	8951	-5	72245	6976
10	5	14403	25	31614	63578
11	8	7754	-40	48722	34218
12	6	7528	73	80376	21440
13	5	7999	-62	30833	12026
14	3	2839	8	44741	7379
15	6	4219	-15	65552	10168
16	4	5186	-10	39927	15279
17	7	7075	23	43477	43949
18	6	4873	-30	77284	15054
19	5	6495	-26	43009	17960
20	8	10274	60	55867	57137

The result of model (9) is given in Table 5. The second and third column shows the optimal value $1/h^*$ from model (3) when client is assumed as an input and an output respectively. The results of model (9) - where client is considered as a flexible measure- are pictured in the fourth column. Likewise, the fifth column illustrates the optimal z.

Table 5. *Results of models (3) and (9) where client is assumed as a flexible measure*

#DMU	Input	Output	Flexible	Z
1	1	1	1	0 or 1
2	0.961446015	1	0.961446015	1
3	0.917767988	1	0.917767988	1
4	0.598300826	0.73964497	0.598300826	1
5	1	1	1	0 or 1
6	0.646872372	0.712656784	0.646872372	1
7	0.666666667	1	0.666666667	1
8	0.866250866	0.99970009	0.866250866	1
9	1	1	1	0 or 1
10	1	1	1	0 or 1
11	1	0.549843295	0.549843295	0
12	0.924556213	1	0.924556213	1
13	0.502638854	0.433350667	0.433350667	0
14	1	1	1	0 or 1
15	1	0.960245823	0.960245823	0
16	1	0.73800738	0.73800738	0
17	0.781433148	1	0.781433148	1
18	1	0.977230529	0.977230529	0
19	1	0.622354991	0.622354991	0
20	0.910663874	1	0.910663874	1

Having a glance at Table 5 reveals that 5 DMUs can treat client as either an input or an output without change in results while 9 out of 15 remaining DMUs consider client as an input. We conclude that, 6 DMUs treat it as an output. Also, according to the majority choice, the flexible measure is identified as an input.

5. Conclusions

The conventional data envelopment analysis assumes that measure status from the viewpoint of input or output is known. However, in real world there are some performance measures that treat as both an input and output. Furthermore, in some situations there exist positive and negative variables. This paper has introduced a model for evaluating the performance of decision making units where variables can take positive or/and negative data and when flexible measures exist. In addition several illustrating examples have been provided to the readers.

6. References

1. Charnes A, Cooper W.W and Rhodes E (1978) Measuring the efficiency of the decision-making units, *European Journal of Operational Research,* 2, 429-444.

2. Cooper W.W, Seiford L.M and Tone K (2000) Data Envelopment Analysis: A Comprehensive Text with models, Applications, References and DEA-solver software, Kluwer Academic Publisher, Boston.

3. Emrouznejad A, Anouze A.L and Thanassoulis E (2010a)A semi-oriented radial measure for measuring the efficiency of decision making units with negative data , using DEA. *European Journal of Operational Research, 200, 297-304.*

4. Cook W. D and Zhu. J (2007) Classifying inputs and outputs in data envelopment analysis. *European Journal of operational Research*. 80, 692-699.

5. Scheel H (2001) undesirable outputs in efficiency valuations, *European Journal of operational Research*. 132, 400-410.

6. Portela M.C.A.S, Thanassoulis E and Simpson G. (2004) A directional distance approach to deal with negative data in DEA, *Journal of Operational Research Society*. 55(10)**,** 1111-1121.

7. Sharp, J.A, Liu, W.B and Meng W (2006) Modified slacks-based measure model for data envelopment analysis with 'natural' negative outputs and inputs, *Journal of Operational Research Society.* 57(11), 1-6.

8. Kazemi Matin R and Azizi, R (2011) A two-phase approach for setting targets in DEA with negative data, *Applied Mathematical Modeling*. 35, 5794-5803.

9.Chinneck, J.W (2004) Practical Optimization: A Gentle introduction,http://www.sce.carleton .ca/faculty/chinneck/po.html.

10. Emrouznejad A, Amin Gh.R, Thanassoulis, E and Anouze A.L (2010b) On the boundedness of the SORM DEA models with negative data, *European Journal of Operational Research*. 206, 265-268.

11. Toloo, M (2009) On classifying inputs and outputs in DEA: a revised model, *European Journal of Operational Research* 198, 358–360.

12. Amirteimoori A and Emrouznejad, A (2012) Notes on ‘‘Classifying inputs and outputs in data envelopment analysis’’, *Applied Mathematics Letters* 25, 1625–1628.

13. Amirteimoori, A and Emrouznejad, A (2011) Flexible Measure In Production Process: A DEA-Based Approach, Radio Operations Research DOI: 10. 1051/ro/ 2011103.

14. Toloo, M (2012) Alternative solutions for classifying inputs and outputs in dea envelopment analysis, *Computers and Mathematics with Applications* 63, 1104-1110.

15 Hadad . M. D. , Hall M. J.B, Kenjegalieva K.A, Santoso W and Simper R (2012) A new approach to dealing with negative numbers in efficiency analysis: An application to the Indonesian banking sector, *Expert Systems with Applications* 39, 8212–8219.

16. Banker R, Charnes A and Cooper W.W (1984) Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management science*. 30, 1078-1092.