Indian Journal of Science and Technology Vol. 3 No. 8 (Aug 2010) ISSN: 0974- 6846

Oscillatory MHD free convective flow through a porous medium with mass transfer, Soret effect and chemical reaction

N.Ahmed and H. Kalita
Dept. of Mathematics, Gauhati University, Guwahati 781014, Assam, India.
saheel_nazib@yahoo.com

Abstract

An attempt is made to investigate the problem of an oscillatory MHD free convective flow through a porous medium with mass transfer, Soret effect and chemical reaction when the temperature as well as concentration at the plate varies periodically with time about a steady mean. Analytical solutions to the coupled non-linear equations governing the flow and heat and mass transfer are obtained by using regular perturbation technique. The influence of the different parameters entering in to the problem viz. the Hartmann number M, the Grashof number for heat transfer G_r , the Grashof number for mass transfer G_m , Soret number S_0 , the plate velocity U , chemical reaction C_h etc. on temperature distribution, species concentration, velocity distribution, skin-friction and the rates of heat and mass transfer at the plate are discussed graphically.

2000 Mathematics subject classification: 76 D, 76 W 05

Keywords: Free convection, MHD, thermal diffusion, chemical reaction.

Nomenclature

A is the suction parameter; $\vec{B}$ is the magnetic induction vector; B_0 is strength of the applied magnetic fluid; $\overline{C}$ is the species concentration of the fluid; C_h chemical reaction parameter; C_p is the specific heat at constant pressure; $\overline{C}_W$ is the species concentration of the fluid at the plate; $\overline{C}_\infty$ is the species concentration far away from the plate; D_M is the coefficient of chemical molecular diffusivity; D_T is the coefficient of chemical thermal diffusivity; E is the Eckert number; G_m is the Grashof number for mass transfer; G_r is the Grashof number for heat transfer; g is the acceleration due to gravity; K is the permeability of the porous medium; k is the thermal conductivity; M is the Hartmann number; P is the Prandtl number; Q is the heat source parameter; S is the Schmidt number; S_0 is the Soret number; $\overline{T}$ is the fluid temperature; $\overline{T}_W$ is the temperature of the fluid at the plate; $\overline{T}_\infty$ is the fluid temperature far away from the plate; t is the time; U is the non-dimensional plate velocity; $\overline{U}$ is the dimensional plate velocity; u is the x component of the non dimensional fluid velocity in the boundary layer; v_0 is the mean suction velocity; $(\overline{u}, \overline{v}, 0)$ are the components of the fluid velocity; y is the non dimensional distance from the plate; $(\overline{x}, \overline{y}, \overline{z})$ are the Cartesian coordinates.

Greek symbols

α is the heat source strength; β is the coefficient of volume expansion for heat transfer; $\overline{\beta}$ is the coefficient of volume expansion for mass transfer; ρ is the fluid density; μ is the coefficient of viscosity; υ is the kinematic viscosity; σ is the electrical conductivity; θ is the non dimensional temperature; ϕ is the non dimensional species concentration; ω is the frequency parameter; ε is the small reference parameter; ξ is the coefficient first order chemical reaction; λ is the reciprocal of the Soret number.

Introduction

The MHD free convective flow and heat transfer problems through porous medium have attracted the attention of a number of scholars due to its importance in many branches of science and technology such as fiber and granular insulations, geothermal system etc. In engineering, its application has been found in MHD pumps, MHD bearing etc. Convection in porous media is applied in geothermal energy recovery, oil extraction; thermal energy storage and flow throw filtering devices. The phenomena of mass transfer are also very common in the theory of stellar structure and in chemical engineering in particular.

In many times, it has been observed that the foreign mass reacts with the fluid and in such a situation chemical reaction plays an important role in chemical industry. Many researchers carried out the study of free convective effects on flow past a vertical surface with different boundary conditions (Vedhanayagam *et al.*, 1980; Kolar & Sastri, 1988; Camargo *et al.*, 1996; Ahmed & Kalita, 2008). The problems of natural convection flow through porous medium past a plate were investigated by Kim & Vafai (1989) and Harris & Ingham (1997).The combined heat and mass transfer effect on MHD free convective flow through porous media was investigated by (Chaudhary & Jain, 2007).

However in the above mentioned works, the thermal-diffusion (Soret) effect was not taken into account. This assumption is justified when the concentration level is very low. The flux of mass caused due to temperature gradient is known as the Soret effect or the thermal-diffusion effect. The experimental investigation of the thermal-diffusion effect on mass transfer related problems was first done by Charles Soret in 1879. There after this thermal-diffusion is termed as the Soret effect in honour of Charles Soret. In general the Soret effect is of a smaller order of magnitude than the effect described in Fick's law and very often it is neglected in mass transfer process. Though this effect is quite small, but the devices can be arranged to produce very steep temperature gradient so that the separation of components in mixtures is affected. Eckert and Drake (1972) have emphasized that in the cases concerning isotope separation and in mixtures between gases with very light molecular weight (H_2, H_e) and for medium molecular weight (N_2, air), Soret effect is found to be of considerable magnitude such that it can not be ignored. Following Eckert and Drake's (1972) work several other investigators have carried out model studies on the Soret and Dufour effects in different heat and mass transfer problems (Dursunkaya & Worek, 1992; Kafoussias & Williams,1995; Sattar & Alam, 1994; Alam *et al.,* 2006; Raju *et al.,*2008).

Ahmed and Kalita (2009a) investigated the effect of the thermal diffusion as well as magnetic field on free convection and mass transfer flow through porous medium, taking into account the effect of a of heat source. Recently Ahmed and Kalita (2009b) have investigated the Soret and magnetic field effects on a transient free convection flow through a porous medium bounded by a uniformly moving infinite vertical porous plate in presence of a heat source.

The aim of the present work is to investigate the problem of an oscillatory MHD free convective flow past a uniformly moving infinite vertical porous plate in a porous medium of Brinkman model (1947) with mass transfer, heat source, Soret effect and chemical reaction. This investigation is an extension of the work done by Ahmed and Kalita to take into account the effect of chemical reaction on the fluid and transport characteristics.

Mathematical formulation

We now consider an unsteady free convective two-dimensional flow with mass transfer of an electrically conducting, viscous and incompressible fluid past an infinite non-conducting vertical plate through a porous medium bounded by an infinite vertical porous plate by making the following assumptions:

(i) The plate is subjected to a normal periodic suction velocity.

(ii) All the fluid properties except the density in the buoyancy force term are constant.

(iii) The Eckert number E is small.

(iv) A uniform magnetic field of strength B_0 is applied transversely to the direction of the main flow.

(v) Magnetic dissipation of energy is negligible.

(vi) The chemical reaction is of first order.

(vii) The magnetic Reynolds number is so small that the induced magnetic field can be neglected.

We consider the flow in the X-direction, which is taken along the length of the plate (vertically upwards) and Y-axes is perpendicular to it directed into the fluid region. Let $\vec{q} = \hat{i}\bar{u} + \hat{j}\bar{v}$ be the fluid velocity at the point $(\bar{x}, \bar{y}, \bar{z})$ and $\vec{B} = B_0\hat{j}$ be the applied magnetic field with strength B_0 , $\hat{i}, \hat{j}$ being the unit vectors along X-axis and Y-axis, respectively. As the plate is of infinite length in X-direction, so all the physical quantities except possibly the pressure p are assumed to be independent of $\bar{x}$. Under these assumptions, the physical quantities are functions of $\bar{y}$ and $\bar{t}$ only.

With the foregoing assumptions, Boussinesq's approximation and under the usual boundary layer approximations the equations governing the flow and transport characteristics are

Equation of continuity:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

Which is trivially satisfied by $\bar{v} = -v_0\left(1 + \varepsilon A e^{i\bar{\omega}\bar{t}}\right)$ (2.1)

Where A is a positive constant such that $\varepsilon A < 1$ and negative sign indicates that the suction velocity is away from the fluid region.

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T}-\bar{T}_\infty) + g\bar{\beta}(\bar{C}-\bar{C}_\infty) + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho}\bar{u} - \frac{\upsilon \bar{u}}{\bar{K}} \quad (2.2)$$

Energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{C_p\rho}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + Q(\bar{T}_\infty - \bar{T}) + \frac{\upsilon}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \quad (2.3)$$

Species continuity equation:

$$\frac{\partial \bar{C}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{C}}{\partial \bar{y}} = D_M\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_T\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + (\bar{C}_\infty - \bar{C})\xi \quad (2.4)$$

The relevant boundary conditions are

$$\left.\begin{array}{l} \bar{y}=0: \bar{u}=\bar{U}, \bar{T}=\bar{T}_W+\varepsilon(\bar{T}_W-\bar{T}_\infty)e^{i\bar{\omega}\bar{t}}, \bar{C}=\bar{C}_W+\varepsilon(\bar{C}_W-\bar{C}_\infty)e^{i\bar{\omega}\bar{t}} \\ \bar{y}\to\infty: \bar{u}\to 0, \bar{T}\to\bar{T}_\infty, \bar{C}\to\bar{C}_\infty \end{array}\right\} \quad (2.5)$$

(2.5)

We now introduce the following non-dimensional quantities:

$$y = \frac{\bar{y}v_0}{\upsilon}, \quad t = \frac{\bar{t}v_0^2}{\upsilon}, \quad \omega = \frac{\upsilon\bar{\omega}}{v_0^2}, \quad u = \frac{\bar{u}}{v_0}, \quad U = \frac{\bar{U}}{v_0},$$

$$\theta = \frac{\bar{T}-\bar{T}_\infty}{\bar{T}_W-\bar{T}_\infty}, \quad \phi = \frac{\bar{C}-\bar{C}_\infty}{\bar{C}_W-\bar{C}_\infty},$$

$$G_r = \frac{g\beta\upsilon(\bar{T}_W-\bar{T}_\infty)}{v_0^3}, \quad G_m = \frac{g\bar{\beta}\upsilon(\bar{C}_W-\bar{C}_\infty)}{v_0^3},$$

$M=\frac{\sigma B_0^2\upsilon}{\rho v_0^2}$, $\alpha=\frac{Q\upsilon}{v_0^2}$, $E=\frac{v_0^2}{C_P(\bar{T}_W-\bar{T}_\infty)}$ $P=\frac{\mu C_p}{k}$,

$K=\frac{\bar{K}v_0^2}{\upsilon^2}$, $S=\frac{\upsilon}{D_M}$, $\lambda=\frac{\upsilon(\bar{C}_W-\bar{C}_\infty)}{D_T(\bar{T}_W-\bar{T}_\infty)}$,

$S_0=\frac{1}{\lambda},\upsilon=\frac{\mu}{\rho},C_h=\frac{S\xi}{v_0^2}D_M$.

All the physical variables are defined in the Nomenclature The dimensionless forms of the equations (2.2), (2.3) and (2.4) are respectively as follows:

$$\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{i\omega t})\frac{\partial u}{\partial y}=G_r\theta+G_m\phi+\frac{\partial^2 u}{\partial y^2}-\left(M+\frac{1}{K}\right)u \quad (2.6)$$

$$P\frac{\partial\theta}{\partial t}-P(1+\varepsilon Ae^{i\omega t})\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial y^2}-P\alpha\theta+PE\left(\frac{\partial u}{\partial y}\right)^2 \quad (2.7)$$

$$S\frac{\partial\phi}{\partial t}-S(1+\varepsilon Ae^{i\omega t})\frac{\partial\phi}{\partial y}=\frac{\partial^2\phi}{\partial y^2}+\frac{S}{\lambda}\frac{\partial^2\theta}{\partial y^2}-S\phi C_h \quad (2.8)$$

The corresponding boundary conditions (2.5) in non-dimensional forms are:

$$\left.\begin{array}{l} y=0:u=U,\theta=1+\varepsilon e^{i\omega t},\phi=1+\varepsilon e^{i\omega t}\\ y\to\infty:u\to 0,\theta\to 0,\phi\to 0\end{array}\right\} \quad (2.9)$$

Method of solution

Assuming the amplitude of oscillation ε to be small, we represent the velocity u, temperature θ and species concentration ϕ near the plate as

$$\left.\begin{array}{l} u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y)+O(\varepsilon^2)\\ \theta(y,t)=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y)+O(\varepsilon^2)\\ \phi(y,t)=\phi_0(y)+\varepsilon e^{i\omega t}\phi_1(y)+O(\varepsilon^2)\end{array}\right\} \quad (3.1)$$

Substituting (3.1) in equations (2.6) to (2.8) and equating the coefficients of ε^0 and ε^1 and neglecting those of ε^2 and higher powers, the following differential equations are obtained:

$$u_0''+u_0'-\left(M+\frac{1}{K}\right)u_0=-G_r\theta_0-G_m\phi_0 \quad (3.2)$$

$$u_1''+u_1'-\left(M+\frac{1}{K}+i\omega\right)u_1=-Au_0'-G_r\theta_1-G_m\phi_1 \quad (3.3)$$

$$\theta_0''+P\theta_0'-P\alpha\theta_0=-EPu_0'^2 \quad (3.4)$$

$$\theta_1''+P\theta_1'-(\alpha+i\omega)P\theta_1=-PA\theta_0'-2PEu_0'u_1' \quad (3.5)$$

$$\phi_0''+S\phi_0'-SC_h\phi_0=-\frac{S}{\lambda}\theta_0'' \quad (3.6)$$

$$\phi_1''+S\phi_1'-S(C_h+i\omega)\phi_1=-SA\phi_0'-\frac{S}{\lambda}\theta_1'' \quad (3.7)$$

subject to the following boundary conditions:

$$\left.\begin{array}{l} y=0:u_0=U,u_1=0;\theta_0=1,\theta_1=1;\phi_0=1,\phi_1=1\\ y\to\infty:u_0\to 0,u_1\to 0;\theta_0\to 0,\theta_1\to 0;\phi_0\to 0,\phi_1\to 0\end{array}\right\} \quad (3.8)$$

where dashes denote differentiation with respect to y. The equations (3.2) to (3.7) are still coupled for the variables u_0, u_1, θ_0, θ_1, ϕ_0 and ϕ_1.

To solve them we note that $E\langle 1$ for all incompressible fluids and we assume that

$$\left.\begin{array}{l} u_0(y)=u_{00}(y)+Eu_{01}(y)+O(E^2);u_1(y)=u_{10}(y)+Eu_{11}(y)+O(E^2)\\ \theta_0(y)=\theta_{00}(y)+E\theta_{01}(y)+O(E^2);\theta_1(y)=\theta_{10}(y)+E\theta_{11}(y)+O(E^2)\\ \phi_0(y)=\phi_{00}(y)+E\phi_{01}(y)+O(E^2);\phi_1(y)=\phi_{10}(y)+E\phi_{11}(y)+O(E^2)\end{array}\right\} \quad (3.9)$$

Substituting from (3.9) in equations (3.2) to (3.7) and by equating the coefficients of E^0 and E^1 in each of the equations and neglecting those of E^2 and higher powers, we obtain the following second order ordinary differential equations:

$$u_{00}''+u_{00}'-\left(M+\frac{1}{K}\right)u_{00}=-G_r\theta_{00}-G_m\phi_{00} \quad (3.10)$$

$$u_{01}''+u_{01}'-\left(M+\frac{1}{K}\right)u_{01}=-G_r\theta_{01}-G_m\phi_{01} \quad (3.11)$$

$$u_{10}''+u_{10}'-\left(M+\frac{1}{K}+i\omega\right)u_{10}=-Au_{00}'-G_r\theta_{10}-G_m\phi_{10} \quad (3.12)$$

$$u_{11}''+u_{11}'-\left(M+\frac{1}{K}+i\omega\right)u_{11}=-Au_{01}'-G_r\theta_{11}-G_m\phi_{11} \quad (3.13)$$

$$\theta_{00}''+P\theta_{00}'-P\alpha\theta_{00}=0 \quad (3.14)$$

$$\theta_{01}''+P\theta_{01}'-P\alpha\theta_{01}=-Pu_{00}'^2 \quad (3.15)$$

$$\theta_{10}''+P\theta_{10}'-(\alpha+i\omega)P\theta_{10}=-AP\theta_{00}' \quad (3.16)$$

$$\theta_{11}''+P\theta_{11}'-(\alpha+i\omega)P\theta_{11}=-AP\theta_{01}'-2Pu_{00}'u_{10}' \quad (3.17)$$

$$\phi_{00}''+S\phi_{00}'-SC_h\phi_{00}=-\frac{S}{\lambda}\theta_{00}'' \quad (3.18)$$

$$\phi_{01}''+S\phi_{01}'-SC_h\phi_{01}=-\frac{S}{\lambda}\theta_{01}'' \quad (3.19)$$

$$\phi_{10}''+S\phi_{10}'-S(C_h+i\omega)\phi_{10}=-SA\phi_{00}'-\frac{S}{\lambda}\theta_{10}'' \quad (3.20)$$

$$\phi_{11}''+S\phi_{11}'-S(C_h+i\omega)\phi_{11}=-SA\phi_{01}'-\frac{S}{\lambda}\theta_{11}'' \quad (3.21)$$

The boundary conditions (3.8) reduce to

$$y=0$$

$$\left.\begin{array}{l} u_{00}=U,u_{01}=0;u_{10}=0,u_{11}=0;\theta_{00}=1,\theta_{01}=0;\theta_{10}=1,\theta_{11}=0\\ \phi_{00}=1,\phi_{01}=0;\phi_{10}=1,\phi_{11}=0\end{array}\right\} \quad (3.22)$$

Indian Journal of Science and Technology Vol. 3 No. 8 (Aug 2010) ISSN: 0974- 6846

$y \to \infty$:

$$\left.\begin{array}{l} u_{00}=0, u_{01}=0; u_{10}=0, u_{11}=0; \theta_{00}=0, \theta_{01}=0; \theta_{10}=0, \theta_{11}=0 \\ \phi_{00}=0, \phi_{01}=0; \phi_{10}=0, \phi_{11}=0 \end{array}\right\} \quad (3.23)$$

Solving the equations from (3.10) to (3.21) subject to the boundary conditions (3.22) and (3.23) we get

$$\theta_{00}(y) = e^{-A_1 y} \quad (3.24)$$

$$\phi_{00}(y) = A_4 e^{-A_2 y} + A_3 e^{-A_1 y} \quad (3.25)$$

$$u_{00}(y) = A_{10} e^{-A_5 y} + A_9 e^{-A_1 y} + A_7 e^{-A_2 y} \quad (3.26)$$

$$\theta_{01}(y) = A_{21} e^{-A_{11} y} + A_{15} e^{-2A_5 y} + A_{16} e^{-2A_1 y} + A_{17} e^{-2A_2 y} + A_{18} e^{-(A_1+A_5)y} + A_{19} e^{-(A_1+A_2)y} + A_{20} e^{-(A_2+A_5)y} \quad (3.27)$$

$$\phi_{01}(y) = A_{36} e^{-A_2 y} + A_{29} e^{-A_{11} y} + A_{30} e^{-2A_5 y} + A_{31} e^{-2A_1 y} + A_{32} e^{-2A_2 y} + A_{33} e^{-(A_1+A_5)y} + A_{34} e^{-(A_1+A_2)y} + A_{35} e^{-(A_2+A_5)y} \quad (3.28)$$

$$u_{01}(y) = B_9 e^{-A_5 y} + B_2 e^{-A_{11} y} + B_3 e^{-2A_5 y} + B_4 e^{-2A_1 y} + B_5 e^{-2A_2 y} + B_6 e^{-(A_1+A_5)y} + B_7 e^{-(A_1+A_2)y} + B_8 e^{-(A_2+A_5)y} + A_{44} e^{-A_2 y} \quad (3.29)$$

$$\theta_{10}(y) = B_{12} e^{-B_{10} y} + B_{11} e^{-A_1 y} \quad (3.30)$$

$$\phi_{10}(y) = B_{19} e^{-B_{13} y} + B_{14} e^{-A_2 y} + B_{18} e^{-A_1 y} + B_{16} e^{-B_{10} y} \quad (3.31)$$

$$u_{10}(y) = B_{33} e^{-B_{20} y} + B_{21} e^{-A_5 y} + B_{30} e^{-A_1 y} + B_{31} e^{-A_2 y} + B_{32} e^{-B_{10} y} + B_{26} e^{-B_{13} y} \quad (3.32)$$

$$\theta_{11}(y) = D_{27} e^{-B_{10} y} + B_{34} e^{-A_{11} y} + D_{21} e^{-2A_5 y} + D_{22} e^{-2A_1 y} + D_{23} e^{-2A_2 y} + D_{24} e^{-(A_1+A_5)y} + D_{25} e^{-(A_1+A_2)y} + D_{26} e^{-(A_2+A_5)y} + D_6 e^{-(A_5+B_{20})y} + D_{10} e^{-(A_5+B_{10})y} + D_{11} e^{-(A_5+B_{13})y} + D_{12} e^{-(A_1+B_{20})y} + D_{15} e^{-(A_1+B_{10})y} + D_{16} e^{-(A_1+B_{13})y} + D_{17} e^{-(A_2+B_{20})y} + D_{19} e^{-(A_2+B_{10})y} + D_{20} e^{-(A_2+B_{13})y} \quad (3.33)$$

$$\phi_{11}(y) = E_{27} e^{-B_{13} y} + D_{28} e^{-A_2 y} + E_{20} e^{-A_{11} y} + E_{21} e^{-2A_5 y} + E_{22} e^{-2A_1 y} + E_{23} e^{-2A_2 y} + E_{24} e^{-(A_1+A_5)y} + E_{25} e^{-(A_1+A_2)y} + E_{26} e^{-(A_2+A_5)y} + E_3 e^{-B_{10} y} + E_{11} e^{-(A_5+B_{20})y} + E_{12} e^{-(A_5+B_{10})y} + E_{13} e^{-(A_5+B_{13})y} + E_{14} e^{-(A_1+B_{20})y} + E_{15} e^{-(A_1+B_{10})y} + E_{16} e^{-(A_1+B_{13})y} + E_{17} e^{-(A_2+B_{20})y} + E_{18} e^{-(A_2+B_{10})y} + E_{19} e^{-(A_2+B_{13})y} \quad (3.34)$$

$$u_{11}(y) = F_{41} e^{-B_{20} y} + E_{28} e^{-A_5 y} + F_{23} e^{-A_{11} y} + F_{24} e^{-2A_5 y} + F_{25} e^{-2A_1 y} + F_{26} e^{-2A_2 y} + F_{27} e^{-(A_1+A_5)y} + F_{28} e^{-(A_1+A_2)y} + F_{29} e^{-(A_2+A_5)y} + F_{30} e^{-A_2 y} + F_{31} e^{-B_{10} y} + F_{32} e^{-(A_5+B_{20})y} + F_{33} e^{-(A_5+B_{10})y} + F_{34} e^{-(A_5+B_{13})y} + F_{35} e^{-(A_1+B_{20})y} + F_{36} e^{-(A_1+B_{10})y} + F_{37} e^{-(A_1+B_{13})y} + F_{38} e^{-(A_2+B_{20})y} + F_{39} e^{-(A_2+B_{10})y} + F_{40} e^{-(A_2+B_{13})y} + F_4 e^{-B_{13} y} \quad (3.35)$$

The constants involved in the solutions are obtained but not presented here for the sake of brevity.

Now substituting equations (3.24) to (3.35) in equation (3.1) and splitting into real and imaginary parts and taking the real parts only we get the expressions for the temperature , species concentration and velocity profiles as follows :

$$\theta(y,t) = \theta_0(y) + \varepsilon(L_r \cos\omega t - L_i \sin\omega t) \quad (3.36)$$

$$\phi(y,t) = \phi_0(y) + \varepsilon(M_r \cos\omega t - M_i \sin\omega t) \quad (3.37)$$

$$u(y,t) = u_0(y) + \varepsilon(N_r \cos\omega t - N_i \sin\omega t) \quad (3.38)$$

Where L_r =Real part of θ_1, L_i =Imaginary part of θ_1, M_r =Real part of ϕ_1, M_i =Imaginary part of ϕ_1, N_r =Real part of u_1, N_i =Imaginary part of ϕ_1

Skin friction

The shear stress distribution is given by

$$\bar{\tau} = \mu \frac{\partial \bar{u}}{\partial \bar{y}} \quad (4.1)$$

The skin-friction in the non-dimensional form on the plate $y = 0$ in the direction of free stream is given by

$$\tau = \left.\frac{\mu \frac{\partial \bar{u}}{\partial \bar{y}}}{\rho v_0^2}\right]_{\bar{y}=0} = \left.\frac{\partial u}{\partial y}\right]_{y=0} = \left\{\frac{\partial u_0}{\partial y} + \varepsilon e^{i\omega t} \frac{\partial u_1}{\partial y}\right\}_{y=0} = u_0^{/}(0) + \varepsilon e^{i\omega t} u_1^{/}(0) \quad , (4.2)$$

Equation (4.1) can be broken up into real and imaginary parts and taking the real part only, we get the skin friction τ as

$$\tau = \tau_0 + \varepsilon |B| cos(\omega t + \gamma) \quad (4.3)$$

Where $B = B_r + iB_i = u_1^{/}(0)$ (4.4)

$$\tau_0 = u_0^{/}(0) \quad (4.5)$$

$$|B| = \sqrt{B_r^2 + B_i^2} \quad (4.6)$$

$$\tan\gamma = \frac{B_i}{B_r} \quad (4.7)$$

Coefficient of the rate of heat transfer

The rate of heat transfer between the fluid and the plate in terms of the Nusselt number is given by

$$Nu = \frac{\upsilon}{v_0(\bar{T}_w - \bar{T}_\infty)} \left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \theta_0^{\prime}(0) + \varepsilon e^{i\omega t} \theta_1^{\prime}(0) \quad (5.1)$$

Splitting the equation (5.1) into real and imaginary parts and taking the real part only, we get

$$Nu = Nu_0 + \varepsilon |H| \cos(\omega t + \eta) \quad (5.2)$$

Where $Nu_0 = \theta_0^{/}(0)$ (5.3)

$$H = H_r + iH_i = \theta_1^{/}(0) \quad (5.4)$$

$$|H| = \sqrt{H_r^2 + H_i^2} \quad (5.5)$$

$$\tan\eta = \frac{H_i}{H_r} \quad (5.6)$$

The expressions for τ_0 , Nu_0 , B_r , B_i , H_r and H_i are obtained but not presented here for the sake of brevity.

Discussion

In order to discuss the effects of various parameters on the flow and transport characteristics near the plate, we have carried out numerical calculations for, θ , ϕ ,u, τ and Nu which are respectively the non-dimensional temperature field, concentration field, velocity field, skin friction at the plate and the Nusselt number at the plate for different values of C_h , U, S_0 ,M, keeping the values of α ,ω, E, ε , ωt , G_r , G_m ,K and A fixed at1,1,0.05,0.001, $\frac{\pi}{2}$,5,2,0.2and 0.2 respectively. The value of the Prandtl number P_r is taken to be 0.7 which corresponds to the air at 20^0 C. Since water vapor is used as a diffusing chemical species of most common interest in air, the value of S (Schmidt number) is considered as 0.60. The values of the other physical parameters are chosen arbitrarily.

Fig. 1-4 exhibits the variation of the temperature field θ against y under the influence of chemical reaction, the plate velocity, Soret number and the magnetic field. It is seen from the Fig. 1, 3 & 4 that the temperature is almost negligible for chemical reaction, thermal diffusion and magnetic field. It is observed from fig. 2 that an increase in the plate velocity results in a steady increase in temperature field.

The Fig. 5-8 depicts the change of behaviour of species concentration ϕ against y under the effects of chemical reaction, plate velocity, Soret number and magnetic parameter respectively. Fig.5 and 6 indicate that an increase in chemical reaction or plate velocity causes the species concentration ϕ to decrease. It is observed from Fig. 7 that the concentration ϕ rises due to the Soret effect whereas from figure 8, it is seen that ϕ is not significantly affected by the applied magnetic field. All Fig. 5-8 further indicate that the concentration steadily falls as y increases.

The effects of chemical reaction, plate velocity, thermal diffusion and applied magnetic field on the velocity u against y are presented in Fig. 9-12. We observed that the flow motion is retarded due to application of the transverse magnetic field and under chemical reaction. Whereas the flow is accelerated under the effect of thermal diffusion and plate velocity.

The influence of the chemical reaction, plate velocity, magnetic parameter and Soret number on the skin friction τ at the plate are displayed in Fig. 13-15. From Fig. 13, 14 & 15 it is clear that the magnitude of the skin friction τ increases when chemical reaction, plate velocity and strength of the applied magnetic field are increased.

Fig. 16-18 demonstrate how the rate of heat transfer in terms of Nusselt number is affected by chemical reaction, plate velocity, Soret number and the magnetic field. Fig. 16 and 17 reveal that there is a fall in values of | Nu | due to the chemical reaction, plate velocity. Significant fall in the rate of heat transfer happens because of chemical reaction and plate velocity: whereas from Fig. 18 we notice that | Nu | increases as S_O increases. Further, the rate of heat transfer from the plate to the fluid is reduced under the action of the applied magnetic field.

Conclusions

(i) Concentration of the fluid falls under chemical reaction and plate velocity.
(ii) Fluid motion is accelerated under thermal deffusion.
(iii) There is retardation in the fluid motion under the effect of chemical reaction, plate velocity and magnetic field.
(iv) |τ| increases under the effect of the chemical reaction, plate velocity and magnetic field whereas it decreases under the thermal diffusion effect.
(v) The rate of heat transfer from the plate to the fluid falls under the effect of C_h , U and magnetic field.

Acknowledgement

The authors thank UGC for supporting this work under MRP F.No.36-96/2008 (SR)

References

1. Ahmed N and Kalita D (2008) MHD oscillatory free convective past a vertical plate in slip-flow regime with variable suction and periodic plate temperature. *Int. J. Heat Technol.* 26 (2), 85-92.
2. Ahmed N and Kalita D (2009a) Effect of thermal diffusion and magnetic field on an unsteady free convective flow with mass transfer through a porous medium in presence of a heat source*. Int. J. Math. Sci. Eng. Appl*. 3(2), 267-281.
3. Ahmed N and Kalita H (2009b) Soret and magnetic field effects on a transient free convection flow through a porous medium bounded by a uniformly moving infinite vertical porous plate in presence of a heat source. *Int. J. Appl. Math. Sci.* in press.
4. Alam S, Rahman MM, Maleque A and Ferdows M (2006) Dufour and Soret effects on steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. *Thammasat Int. J. Sci. Tech.* 11(2), 1-12.
5. Brinkman HC (1947) A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. *Appl. Sci. Res.* A1, 81-86.

6. Camargo R, Luna E and Trevine C (1996) Numerical study of natural convective cooling of a vertical plate. *Heat Mass Transfer.* 32, 89-95.
7. Chaudhary RC and Jain A (2007) Combined heat and mass transfer effects on MHD free convection flow past an oscillatory plate embedded in porous medium. *Rom. J. Phys*. 52 (5-7), 505-524.
8. Durunkya Z and Woret WM (1992) Diffusion-Thermo and thermal diffusion effects in transient and steady natural convection from vertical surface. *Int. J. Heat Mass Transfer.* 35(8), 2060-2065.
9. Eckert ERG and Drake RM (1972) Analysis of heat and mass transfer, Mc Grow-Hill, NY.
10. Harris SD and Ingham DB (1997) Free convection from a vertical plate in a porous medium subject to sudden change in surface heat flux. *Trans. porous media.* 26, 207-226.
11. Kafoussias NG and Williams EM (1995) Thermal-diffusion and diffusion-Thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependant viscosity. *Int. J. Engg. Sci*. 33, 1369-1384.
12. Kim SJ and Vafai V (1989) Analysis of natural convection about a vertical plate embedded in porous medium. *Int. J. Heat Mass Transfer.* 32, 665-677.
13. Kolar AK and Sastri VM (1988) Free convective transpiration over a vertical plate: A numerical study. *Heat Mass Transfer.* 23(6), 327-336.
14. Raju MC, Varma SVK, Reddy PV and Saha S (2008) Soret effects due to natural convection between heated inclined plates with magnetic field. *J. Mech. Engg. ME.* 39(2), 65-70.
15. Sattar MA and Alam MM (1994) Thermal diffusion as well as transpiration effects on MHD free convection and mass transfer flow past an accelerated vertical porous plate. *Ind. J. Pure Appl. Math.* 25(6), 679-688.
16. Vedhanayagam M, Altenkrich RA and Eichhorn R (1980) A transformation of the boundary layer equation for free convection flow past a vertical flat plate with arbitrary blowing and wall temperature variation. *Int. J. Heat Mass Transfer.* 23,1236-1288.