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Hydromagnetic oscillatory flow and heat transfer of a viscous liquid past a vertical porous plate in a rotating medium

S. N. Sahoo, J. P. Panda and G.C. Dash
Dept. of Mathematics, Institute of Technical Education and Research, Siksha 'O' Anusandhan University, Khandagiri Square, Bhubaneswar-751030, Orissa, India
sachi.sahoo@yahoo.com; jpanda123@rediffmail.com; gcdash@indiatimes.com

Abstract

Unsteady oscillatory flow of an incompressible, electrically conducting viscous liquid through a porous medium past an infinite vertical porous plate with constant suction has been studied. A transverse uniform magnetic field has been applied in a rotating frame of reference. The porous medium is bounded by a vertical plane surface. The temperature on the vertical surface fluctuates in time about a non-zero constant mean. The analytical expressions for the velocity, temperature, skin friction and Nusselt number are presented showing the effects of pertinent parameters. It is observed that steady part of the velocity field has two layers character viz thermal layer and suction layer, while the oscillatory part exhibits a multilayer character.

Keywords: Hydromagnetic, oscillatory, heat transfer, porous, rotating system

Introduction

The study of magnetohydrodynamics (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow has wide application in engineering and technology. Jha (1998) has studied the effects of applied magnetic field on transient free convective flow in a vertical channel. Singh *et al.* (2000) have analyzed the natural convection in a non-rectangular porous cavity. Acharya *et al.* (2000) have reported the magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Singh *et al.* (2001) have discussed the free convection in MHD flow of a rotating viscous liquid in porous medium past a vertical porous plate. Israel-Cookey *et al.* (2003) have studied the unsteady MHD free-convection and mass transfer flow past an infinite heated porous vertical plate with time dependant suction. Das *et al.* (2004) have reported free convection flow and mass transfer of an elastico-viscous fluid past an infinite vertical porous plate in a rotating porous medium. Panda *et al.* (2004) have analyzed the unsteady free convection MHD flow and mass transfer in a rotating porous medium.

Kurtcebe and Erim (2005) have reported the heat transfer of a visco-elastic fluid in a porous channel. Ogulu *et al.* (2005) have analyzed the numerical study of mixed convection heat transfer from thermal sources on a vertical surface. Cheng (2006) have analyzed the free convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micropolar fluid. Panda *et al.* (2006) have reported the free Convection of conducting viscous fluid between two vertical walls filled with porous material. Dash *et al.* (2002) have studied the flow of unsteady visco-elastic electrically conducting fluid between two porous concentric circular cylinders. Sengupta and Basak (2002) have analyzed the unsteady flow of visco-elastic maxwell fluid through porous straight tube under uniform magnetic field. Israel-Cookey *et al.* (2003) have analyzed the unsteady MHD free-convection flow past semi-infinite heated porous vertical plate with time-dependent suction and radiative heat transfer. Singh *et al.* (2003) have analyzed heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Sharma and Pareek (2006) have analyzed unsteady flow and heat transfer of an elastico-viscous liquid along an infinite hot vertical porous moving plate with variable free stream and suction. In the above study, we examine the hydromagnetic oscillatory flow and heat transfer of a viscous liquid past a vertical porous plate in a rotating system. The governing equations of the flow field are solved for velocity and temperature of the flow field and effects of the important parameters on the flow field have been analyzed and discussed with the aid of figures and tables.

Formulation of the problem

Consider an oscillatory flow of a viscous incompressible electrically conducting liquid past an infinite vertical porous plate through a porous medium in a rotating system. The x- and y-axes are taken along the plate and z-axis is normal to the plate and the component of velocity $\vec{q}$ in these directions are u, v, w respectively. The plate is porous subjected to suction velocity $w=-w_0$, where w_0 is real and positive. The liquid and the plate both are in a state of rigid body rotation with uniform angular velocity Ω about z-axis. The plate is of infinite extent therefore all the physical variables depend on z and t only. A uniform magnetic field $B_o=\mu_e H$, where H= (0, 0, H_0) has been applied in the z-direction i.e. normal to the flow. The buoyancy force and hall Effect are not considered and the effects due to perturbation of the field have been neglected. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied magnetic field. The free stream velocity is $U = 1+\varepsilon e^{\text{int}}$, where n < < 1. Under these assumptions, the equation of continuity is

$$\frac{\partial w}{\partial z} = 0 \quad (1)$$

The equations of motion and temperature are

$$\frac{\partial u}{\partial t} - w_0\frac{\partial u}{\partial z} - 2\Omega v = \frac{\partial U}{\partial t} + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\nu}{K}(u-U) - \frac{\sigma}{\rho}\mu_e{}^2 H_0{}^2(u-U) \quad (2)$$

$$\frac{\partial v}{\partial t} - w_0\frac{\partial v}{\partial z} + 2\Omega(u-U) = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\nu}{K}v - \frac{\sigma}{\rho}\mu_e{}^2 H_0{}^2 v \quad (3)$$

$$\frac{\partial T}{\partial t} - w_0\frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial z^2} + S(T-T_\infty) \quad (4)$$

where U is the free stream velocity, H_0 is the constant magnetic field, K is the permeability of the medium, S is the source, C_p is the specific heat of the fluid, κ is the thermal conductivity of the fluid and other symbols have their usual meaning.

The boundary conditions relevant to the problem are

$$u = 0, \quad v = 0, \quad T = T_\omega + \varepsilon(T_\omega - T_\infty)e^{\text{int}} \text{ at z = 0.}$$

$$u \to U_0(1+\varepsilon e^{\text{int}}),\ v \to 0,\ T \to T_\infty \text{ as z} \to\infty. \quad (5)$$

We introduce the following non-dimensional quantities:

$$z^* = \frac{w_0 z}{\nu}, t^* = \frac{w_0{}^2 t}{\nu}, U^* = \frac{U}{U_0}, n^* = \frac{\nu n}{w_0{}^2}, T^* = \frac{T-T_\infty}{T_w - T_\infty}$$

$$S^* = \frac{S\nu}{w_0^2}, \quad q^* = \frac{u}{U_0} + i\frac{v}{U_0} = u^* + iv^*$$

$$K_p = \frac{w_0{}^2 K}{\nu^2} \text{ (Permeability parameter)}$$

$$R = \frac{\Omega\nu}{w_0{}^2} \text{ (Rotation parameter)}$$

$$M^2 = \frac{\sigma\mu_e{}^2 H_0{}^2 \nu}{\rho w_0{}^2} \text{ (Magnetic parameter)}$$

and $\Pr = \frac{\mu C_p}{\kappa}$ (Prandtl number)

Using the above stated non-dimensional quantities and neglecting asterisks over them, eqn. (2) and (3) are transformed to eqn. (6) and eqn. (4) is transformed to eqn. (7).Thus we have

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} + (M^2 + \frac{1}{K_p} + 2iR)(q-U) = \frac{\partial U}{\partial t} + \frac{\partial^2 q}{\partial z^2} \quad (6)$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial z} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial z^2} + ST \quad (7)$$

The boundary conditions (4) become

$$q = 0, \quad T = 1+\varepsilon e^{\text{int}} \text{ at z = 0.}$$

$$q = 1+\varepsilon e^{\text{int}}, \quad T = 0 \text{ as z} \to\infty. \quad (8)$$

Solutions of the problem

To solve eqns. (6) and (7), we assume the velocity and temperature of the liquid in the neighbourhood of the plate as

$$q(z,t) = (1-q_0) + \varepsilon(1-q_1)e^{\text{int}} \text{ and}$$

$$T(z,t) = T_0(z) + \varepsilon T_1(z)e^{\text{int}}$$

Using *q and U* in eqn. (6) and equating terms independent of ε and co-efficient of ε, we have

$$\frac{d^2 q_0}{dz^2} + \frac{dq_0}{dz} - (M^2 + \frac{1}{K_p} + 2iR)q_0 = 0 \quad (9)$$

$$\frac{d^2 q_1}{dz^2} + \frac{dq_1}{dz} - (M^2 + \frac{1}{K_p} + 2iR + in)q_1 = 0 \quad (10)$$

The boundary conditions (7) for velocity becomes

$$q_0 = 1, \quad q_1 = 1 \text{ at z = 0.}$$

$$q_0 = 0, \quad q_1 = 0 \quad \text{as z} \to\infty. \quad (11)$$

The solutions of eqns. (9) and (10) in view of (11) are given by

$$q_0(z) = e^{-A_1 z} \quad (12)$$

and $q_1(z) = e^{-A_2 z}$ (13)

Where $A_1 = 0.5\left[1+\sqrt{1+4(M^2+\frac{1}{K_p}+2iR)}\right]$

$$= \alpha_1 + i\beta_1,$$

$$\alpha_1 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2+\frac{4}{K_p})^2 + 64R^2} + (1+4M^2+\frac{4}{K_p})\right]^{\frac{1}{2}}$$

$$\beta_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2+\frac{4}{K_p})^2 + 64R^2} - (1+4M^2+\frac{4}{K_p})\right]^{\frac{1}{2}}$$

$$A_2 = 0.5\left[1+\sqrt{1+4(M^2+\frac{1}{K_p}+2iR+in)}\right]$$

$$= \alpha_2 + i\beta_2,$$

$$\alpha_2 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2+\frac{4}{K_p})^2+(8R+4n)^2}+(1+4M^2+\frac{4}{K_p})\right]^{\frac{1}{2}}$$

$$\beta_2 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M^2+\frac{4}{K_p})^2+(8R+4n)^2}-(1+4M^2+\frac{4}{K_p})\right]^{\frac{1}{2}}$$

Therefore $q(z,t) = (1-q_0) + \varepsilon(1-q_1)e^{\text{int}}$ (14)

$= u + iv,$

Where

$$u = 1 - e^{-\alpha_1 z}\cos\beta_1 z + \varepsilon(1 - e^{-\alpha_2 z}\cos\beta_2 z)\cos nt - \varepsilon e^{-\alpha_2 z}\sin\beta_2 z\sin nt \quad (15)$$

$$v = e^{-\alpha_1 z}\sin\beta_1 z + \varepsilon e^{-\alpha_2 z}\sin\beta_2 z\cos nt + \varepsilon(1 - e^{-\alpha_2 z}\cos\beta_2 z)\sin nt \quad (16)$$

Skin-friction

The skin-friction τ in non-dimensional form is given by

$$\tau = \frac{\partial q}{\partial z}\Big]_{z=0} = \alpha_1 + \varepsilon(\alpha_2\cos nt - \beta_2\sin nt) + i\{\beta_1 + \varepsilon(\beta_2\cos nt + \alpha_2\sin nt)\} \quad (17)$$

The transient primary (τ_p) and transient secondary (τ_s) skin-friction at *nt* = π/2 are

$$\tau_p = \frac{du}{dz}\Big]_{z=0} = \alpha_1 - \varepsilon\beta_2 \quad (18)$$

$$\tau_s = \frac{dv}{dz}\Big]_{z=0} = \beta_1 + \varepsilon\alpha_2 \quad (19)$$

Putting T and its corresponding derivatives in eqn. (7) and equating terms independent of ε and coefficient of ε, we have

$$\frac{d^2T_0}{dz^2} + \Pr\frac{dT_0}{dz} + \Pr ST_0 = 0 \quad (20)$$

$$\frac{d^2T_1}{dz^2} + \Pr\frac{dT_1}{dz} + (S - in)\Pr T_1 = 0 \quad (21)$$

The boundary conditions (7) for temperature becomes

$T_0 = 1$, $T_1 = 1$ at $z = 0$

$T_0 = 0$, $T_1 = 0$ as $z \to \infty$ (22)

The solutions of equations (20) and (21) in view of (22) are given by

$$T_0(z) = e^{-P_1 z} \quad (23)$$

and $T_1(z) = e^{-P_2 z}$ (24)

where $P_1 = 0.5\left[\Pr + \sqrt{\Pr^2 - 4\Pr S}\right]$, (25)

$$P_2 = 0.5\left[\Pr + \sqrt{\Pr^2 - 4(S - in)\Pr}\right] = \alpha_3 + i\beta_3,$$

$$\alpha_3 = \frac{\Pr}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(\Pr^2 - 4S\Pr)^2 + 16n^2\Pr^2} + (\Pr^2 - 4S\Pr)\right]^{\frac{1}{2}}$$

$$\beta_3 = \frac{1}{2\sqrt{2}}\left[\sqrt{(\Pr^2 - 4S\Pr)^2 + 16n^2\Pr^2} - (\Pr^2 - 4S\Pr)\right]^{\frac{1}{2}}$$

So, $T(z,t) = e^{-P_1 z} + \varepsilon e^{-P_2 z + \text{int}}$ (26)

$= T_p + iT_s$,

Where $T_p = e^{-P_1 z} + \varepsilon e^{-\alpha_3 z}\sin\beta_3 z$ and $T_s = \varepsilon e^{-\alpha_3 z}\cos\beta_3 z$.

Rate of heat transfer

The rate of heat transfer in terms of Nusselt number at the surface is given by

$$Nu = -\frac{\partial T}{\partial z}\Big]_{z=0} = (Nu)_p + i(Nu)_s$$

Where $(Nu)_p = P_1 + \varepsilon(\alpha_3\cos nt - \beta_3\sin nt)$

and $(Nu)_s = \varepsilon(\alpha_3\sin nt - \beta_3\cos nt)$.

For *nt = π/2*, we have $(Nu)_p = P_1 - \varepsilon\beta_3$

and $(Nu)_s = \varepsilon\alpha_3$.

Results and discussion

Unsteady oscillatory flow of an electrically conducting viscous liquid through a porous medium under the influence of transient velocity and temperature field in a rotating system has been characterized by magnetic parameter (M), rotation parameter (R) and permeability parameter (K_p) with a fluctuating free stream velocity. The main objective of the discussion is to bring out effect of pertinent parameters governing the flow and heat transfer phenomenon. During computation the value of ε=0.05 and n=0.01.

Fig.1 shows parabolic velocity distribution (curve-VIII) with maximum magnitude for high value of K_p(=100) i.e. representing the case of without porous matrix. Due to resistance of porous matrix, velocity decreases. Magnetic field as well as rotating system leads to enhance the primary velocity. When magnetic field and rotating force are withdrawn (i.e.M=0 & R=0), the velocity assumes the lowest value (curves VI & VII). Fig. 2 shows the transient secondary velocity. The nature of the profiles is almost similar to that of primary. But it is interesting to observe the curve-VIII, the case of without porous matrix. Secondary velocity assumes highest value near the plate and attains uniform velocity at the earliest. It is evident from curves VI and VII; the secondary velocity is almost linear. It is important to note that in the absence of

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Fig. 1. Effect of M, R, Kp on primary velocity profiles when n=0.01, ε=0.05

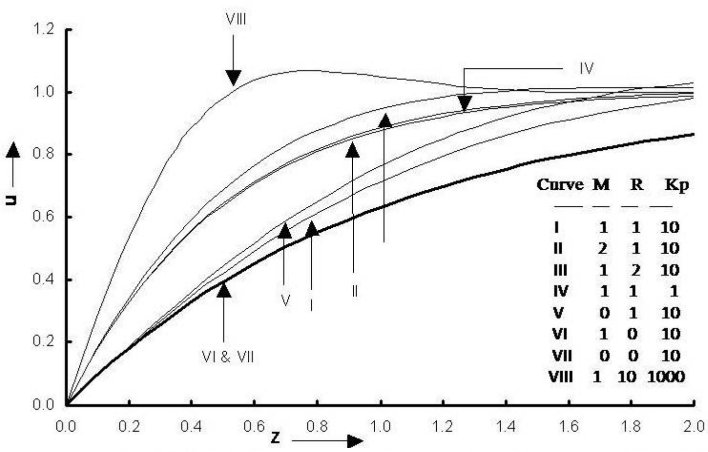

Fig. 2. Effect of M, R, Kp on secondary velocity profiles when n=0.01, ε=0.05

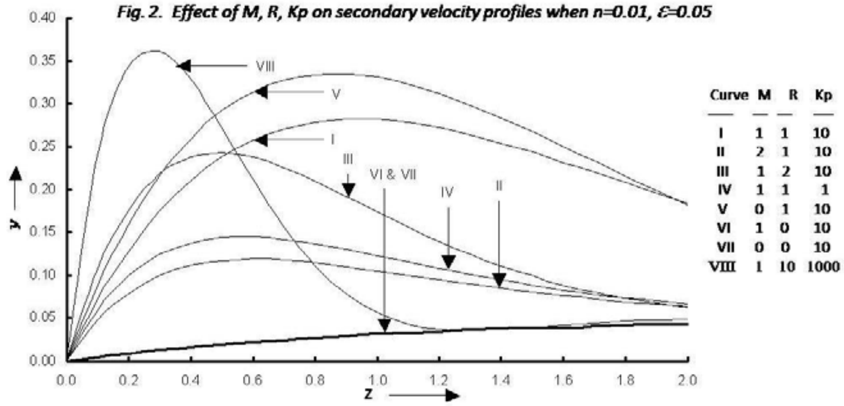

Fig. 3. Effect of Pr, S on temperature profiles

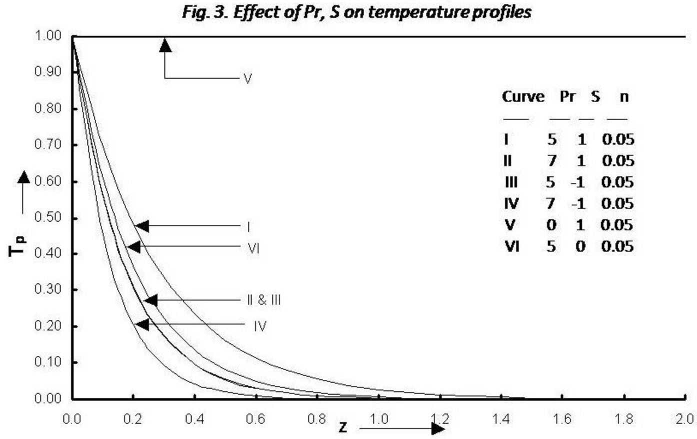

Fig. 4. Effect of Pr, S on temperature profiles

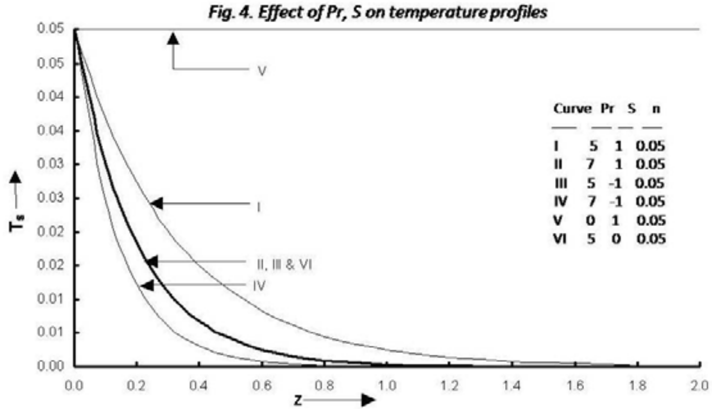

Table 1. Variation in the value of skin friction at the wall against M for different values of R , K_p , n, ε

M	R	K_p	n	ε	τ_p	τ_s
1.0	1.0	10.0	0.01	0.050	0.963406	0.779036
2.0	1.0	10.0	0.01	0.050	1.976497	0.567838
3.0	1.0	10.0	0.01	0.050	2.983660	0.475201
1.0	3.0	10.0	0.01	0.050	1.922462	1.649193
1.0	1.0	20.0	0.01	0.050	0.963024	0.786672
1.0	1.0	30.0	0.01	0.050	0.962895	0.789255
1.0	1.0	10.0	0.02	0.050	0.963265	0.779036
1.0	1.0	10.0	0.05	0.050	0.962842	0.779036
1.0	1.0	10.0	0.01	0.100	0.926813	0.829036
1.0	1.0	10.0	0.01	0.005	0.996341	0.734036
0.0	1.0	10.0	0.01	0.050	0.954035	0.966623
0.0	0.0	1000.0	0.01	0.050	0.999501	0.050000
1.0	1.0	1000.0	0.01	0.050	0.962642	0.794320

Table 2. Variation in the value of rate of heat transfer at the surface against Pr for different values of S , ε , n

Pr	S	ε	n	$(Nu)_p$	$(Nu)_s$
5.0	1.0	0.050	0.05	3.612467303	o.181177785
7.0	1.0	0.050	0.05	5.787471152	0.289627951
5.0	-1.0	0.050	0.05	5.852238692	0.292715448
7.0	-1.0	0.050	0.05	7.885487967	0.394383173
0.0	1.0	0.050	0.05	0.000000000	0.000000000
0.0	-1.0	0.050	0.05	0.000000000	0.000000000
5.0	0.0	0.050	0.05	4.997500500	0.250024987
5.0	1.0	0.100	0.05	3.606904606	0.362355571
5.0	1.0	0.005	0.05	3.617473731	0.018117778
5.0	1.0	0.050	0.01	3.616912190	0.180912874

rotation (R=0, M≠0), the magnetic field has no significant contribution but when R≠0 and M=0 (the curve-V), the secondary velocity attains significantly high value. Thus, it is concluded that contribution of rotation is independent of the effect of magnetic field.

Fig. 3 shows the real part of fluctuating temperature distribution. For high Prandtl number fluid i.e. with dominating viscosity and low conductivity, temperature decreases. The same effect is observed in the presence of sink. But in the presence source, temperature increases. It is interesting to note that the case of Pr=0 i.e. μ=0 i.e. for perfect fluid, temperature distribution assumes a constant value. It is also evident from the eqn. (25). Fig. 4 shows the imaginary part of temperature distribution. The effects of Pr & S are the same as on case of $(Nu)_p$. The nature of distribution is exponential in both the cases.

Skin friction

The numerical values of skin friction remain positive for all the values of the parameter. It is observed that as the magnetic parameter increases, primary velocity experiences increasing skin-friction but the decreasing effect is observed on the secondary one (Table 1). In case of higher values of rotation parameter the skin-frictions corresponding to primary and secondary velocity increase.

Rate of heat transfer

It is observed from the Table 2 that the Nusselt number at the surface increases due to increase in the Prandtl number. The rate of heat transfer is zero when the Prandtl number is zero. As the source strength decreases, the rate of heat transfer increases. It is clear from the eqn. (25) that source strength greater than 2.0 contributes to the fluctuating part.

Conclusion

The above study brings out the following conclusion:

1. The effects of magnetic field and rotation parameter are to enhance the primary velocity.
2. The contribution of rotation parameter R is independent of the magnetic field.
3. The temperature increases in the presence of source.
4. For high Prandtl number fluid, the temperature decreases.
5. The effect of Prandtl number Pr and source parameter S on secondary temperature profile is same as that of primary.

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